

φ153

1. Escribe en forma polar los siguientes n.º complejos:

a) $1 + \sqrt{3}i$

b) $\sqrt{3} + i$

c) $-1 + i$

d) $5 - 12i$

e) $3i$

f) -5

a) $1 + \sqrt{3}i$

$$\left. \begin{aligned} |z| &= \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2 \\ \alpha &= \operatorname{arctg} \frac{\sqrt{3}}{1} = \operatorname{arctg} \sqrt{3} = \frac{\pi}{3} \end{aligned} \right\} \left(2 \frac{\pi}{3} \right)$$

b) $\sqrt{3} + i$

$$\left. \begin{aligned} |z| &= \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2 \\ \alpha &= \operatorname{arctg} \frac{1}{\sqrt{3}} = \frac{\pi}{6} \end{aligned} \right\} \left(2 \frac{\pi}{6} \right)$$

c) $-1 + i$

$$\left. \begin{aligned} |z| &= \sqrt{(-1)^2 + 1^2} = \sqrt{2} \\ \alpha &= \operatorname{arctg} \frac{1}{-1} = \operatorname{arctg} (-1) = -\frac{\pi}{4} \end{aligned} \right\} \left(\sqrt{2} \frac{3\pi}{2} \right) \quad (1^{\text{er}} \text{ cuadrante})$$

d) $5 - 12i$

$$\left. \begin{aligned} |z| &= \sqrt{5^2 + (-12)^2} = \sqrt{25+144} = \sqrt{169} = 13 \\ \alpha &= \operatorname{arctg} \left(-\frac{12}{5} \right) = 112,62^\circ \end{aligned} \right\} \left(13 \frac{292^\circ 37'}{ } \right)$$

e) $3i$

$$\left. \begin{aligned} |z| &= \sqrt{0+3^2} = \sqrt{3^2} = 3 \\ \alpha &= \operatorname{arctg} \frac{3}{0} = \operatorname{arctg} \infty = \frac{\pi}{2} \end{aligned} \right\} \left(3 \frac{\pi}{2} \right)$$

$$1) -5$$

$$|z| = \sqrt{(-5)^2 + 0^2} = \sqrt{25} = 5$$

$$\alpha = \operatorname{arctg} \frac{0}{-5} = \operatorname{arctg} 0 = 0^\circ \quad \left. \vphantom{\alpha = \operatorname{arctg} \frac{0}{-5}} \right\} 50^\circ$$

2, p 153

2.- Escribe en f. binómica los siguientes n.ºs complejos:

a) $5 \frac{4}{6}$

b) $2 \ 135^\circ$

c) $2 \ 495^\circ$

d) $3 \ 240^\circ$

e) $5 \ 180^\circ$

f) $4 \ 90^\circ$

$$a) \ 5 \frac{4}{6} = 5 \left(\cos \frac{4}{6} + i \sin \frac{4}{6} \right) = 5 \left(\frac{\sqrt{3}}{2} + i \frac{1}{2} \right) = \frac{5\sqrt{3}}{2} + \frac{5i}{2}$$

$$b) \ 2 \ 135^\circ = 2 \left(\cos 135^\circ + i \sin 135^\circ \right) = 2 \left(-\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \right) = -\sqrt{2} + \sqrt{2}i$$

$$c) \ 2 \ 495^\circ = 2 \ 135^\circ = 2 \left(\cos 135^\circ + i \sin 135^\circ \right) = 2 \left(-\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \right) = -\sqrt{2} + \sqrt{2}i$$

$$d) \ 3 \ 240^\circ = 3 \left(\cos 240^\circ + i \sin 240^\circ \right) = 3 \left(-\frac{1}{2} - i \frac{\sqrt{3}}{2} \right) = -\frac{3}{2} - \frac{3\sqrt{3}}{2}i$$

$$e) \ 5 \ 180^\circ = 5 \left(\cos 180^\circ + i \sin 180^\circ \right) = 5 \left(-1 + 0 \right) = -5$$

$$f) \ 4 \ 90^\circ = 4 \left(\cos 90^\circ + i \sin 90^\circ \right) = 4 \left(0 + i1 \right) = 4i$$

3, p153

3.- Expresa en f. polar el opuesto y el conjugado del n.º complejo: $z = r \angle \alpha$

OPUESTO: $-z = r \angle 180^\circ + \alpha$

CONJUGADO: $\bar{z} = r \angle 360^\circ - \alpha$

4.- Escribe en f. binómica y en f. polar el complejo:

$$z = 8(\cos 30^\circ + i \sin 30^\circ)$$

$$z = 8(\cos 30^\circ + i \sin 30^\circ) = 8\left(\frac{\sqrt{3}}{2} + i \frac{1}{2}\right) = 4\sqrt{3} + 4i = \frac{8\sqrt{3}}{2} + \frac{8}{2}i$$

f. trigonom. f. binómica f. polar

$$|z| = \sqrt{(4\sqrt{3})^2 + 4^2} = \sqrt{16(3) + 16} = \sqrt{48 + 16} = \sqrt{64} = 8$$

$$\alpha = \arctg \frac{4}{4\sqrt{3}} = \arctg \frac{1}{\sqrt{3}} = \frac{\pi}{6}$$

5.- Sean los n.º complejos: $z_1 = 4_{60^\circ}$ y $z_2 = 3_{210^\circ}$

a) Expresa z_1 y z_2 en f. binómica

b) Halla $z_1 \cdot z_2$ y z_2/z_1 y para los resultados en f. polar.

c) Compara los módulos y los argumentos de $z_1 \cdot z_2$ y z_2/z_1 con los de z_1 y z_2 e intenta encontrar relaciones entre ellos.

$$a) \quad z_1 = 4_{60^\circ} = 4(\cos 60^\circ + i \sin 60^\circ) = 4\left(\frac{1}{2} + i \frac{\sqrt{3}}{2}\right) = 2 + 2\sqrt{3}i$$

$$z_2 = 3_{210^\circ} = 3(\cos 210^\circ + i \sin 210^\circ) = 3\left(-\frac{\sqrt{3}}{2} - i \frac{1}{2}\right) = -\frac{3\sqrt{3}}{2} - \frac{3}{2}i$$

$$b) \quad z_1 \cdot z_2 = (2 + 2\sqrt{3}i) \left(-\frac{3\sqrt{3}}{2} - \frac{3}{2}i\right) = -3\sqrt{3} - 3i - 9i + 3\sqrt{3} = -12i = 12_{270^\circ}$$

COMPROBACIÓN

$$4_{60^\circ} \cdot 3_{210^\circ} = 12_{270^\circ}$$

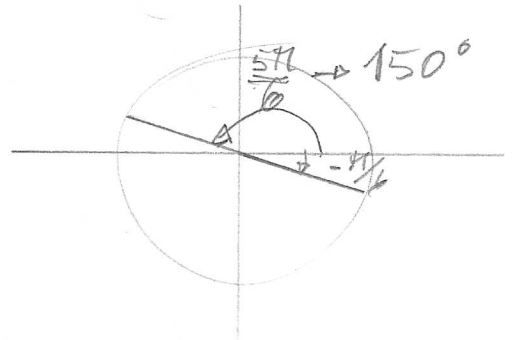
$$\frac{z_2}{z_1} = \frac{\left(-\frac{3\sqrt{3}}{2} - \frac{3}{2}i\right)}{(2+2\sqrt{3}i)} = \frac{\left(-\frac{3\sqrt{3}}{2} - \frac{3}{2}i\right) \cdot (2-2\sqrt{3}i)}{(2+2\sqrt{3}i)(2-2\sqrt{3}i)} =$$

$$= \frac{-3\sqrt{3} + 9i - 3i - 3\sqrt{3}}{4 + 12} = \frac{-6\sqrt{3} + 6i}{16} = -\frac{3\sqrt{3}}{8} + \frac{3}{8}i$$

$$\left|\frac{z_2}{z_1}\right| = \sqrt{\left(-\frac{3\sqrt{3}}{8}\right)^2 + \left(\frac{3}{8}\right)^2} = \sqrt{\frac{27}{64} + \frac{9}{64}} = \sqrt{\frac{36}{64}} = \frac{6}{8} = \frac{3}{4}$$

$$\alpha = \operatorname{arctg} \frac{\frac{3}{8}}{-\frac{3\sqrt{3}}{8}} = \operatorname{arctg}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6} = \frac{5\pi}{6}$$

$$\boxed{\frac{z_2}{z_1} = \left(\frac{3}{4}\right) \frac{5\pi}{6}}$$



COMPROBACIÓN

$$\frac{z_2}{z_1} = \frac{3 \angle 110^\circ}{4 \angle 60^\circ} = \left(\frac{3}{4}\right) \angle 150^\circ$$

c) las comprobaciones

p 155

1.- Efectúa estas operaciones y da el resultado en f. polar y binómica.

a) $1_{150^\circ} \cdot 5_{30^\circ}$

b) $6_{45^\circ} : 3_{15^\circ}$

c) $2_{10^\circ} \cdot 1_{40^\circ} \cdot 3_{70^\circ}$

d) $5_{\frac{2\pi}{3}} : 1_{60^\circ}$

e) $(1 - \sqrt{3}i)^5$

f) $(3+2i) + (-3+2i)$

a) $1_{150^\circ} \cdot 5_{30^\circ} = \boxed{5_{180^\circ}}$
 $5_{180^\circ} = 5 \left(\underset{-1}{\cancel{\cos 180}} + i \underset{0}{\cancel{\sin 180}} \right) = \boxed{-5}$

b) $\frac{6_{45^\circ}}{3_{15^\circ}} = \left(\frac{6}{3} \right)_{30^\circ} = \boxed{2_{30^\circ}}$
 $2_{30^\circ} = 2 \left(\cos 30 + i \sin 30 \right) = 2 \left(\frac{\sqrt{3}}{2} + i \frac{1}{2} \right) = \boxed{\sqrt{3} + i}$

c) $2_{10^\circ} \cdot 1_{40^\circ} \cdot 3_{70^\circ} = \boxed{6_{120^\circ}}$
 $6_{120^\circ} = 6 \left(\cos 120 + i \sin 120 \right) = 6 \left(-\frac{1}{2} + i \frac{\sqrt{3}}{2} \right) =$
 $= \boxed{-3 + 3\sqrt{3}i}$

d) $\frac{5_{\frac{2\pi}{3}}}{1_{60^\circ}} = \frac{5_{120^\circ}}{1_{60^\circ}} = \boxed{5_{60^\circ}}$
 $5_{60^\circ} = 5 \left(\cos 60 + i \sin 60 \right) = 5 \left(\frac{1}{2} + i \frac{\sqrt{3}}{2} \right) = \boxed{\frac{5}{2} + \frac{5\sqrt{3}}{2}i}$

$$e) (1 - \sqrt{3}i)^5 = (2_{300})^5 = (2^5)_{5 \times 300} = \boxed{32_{1500^\circ}}$$

$$\left. \begin{aligned} r &= \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2 \\ \alpha &= \arctan(-\sqrt{3}) = 300^\circ \end{aligned} \right\} 2_{300^\circ}$$

$1500^\circ \leadsto 60^\circ$ y 4 vueltas

$$\begin{aligned} 32_{1500^\circ} &= 32_{60^\circ} = 32(\cos 60^\circ + i \sin 60^\circ) = 32\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) \\ &= \boxed{16 + 16\sqrt{3}i} \end{aligned}$$

$$4) (3+2i) + (-3+2i) = \cancel{3} + 2i - \cancel{3} + 2i = \boxed{4i}$$

$$4i = \boxed{4_{\pi/2}}$$

2.- Comparar los resultados en cada caso:

$$a) (2_{30^\circ})^3, (2_{150^\circ})^3, (2_{270^\circ})^3$$

$$b) (2_{60^\circ})^4, (2_{150^\circ})^4, (2_{270^\circ})^4, (2_{330^\circ})^4$$

$$a) (2_{30^\circ})^3 = (2^3)_{3 \times 30} = 8_{90^\circ}$$

$$(2_{150^\circ})^3 = (2^3)_{3 \times 150} = 8_{450^\circ} = 8_{90^\circ}$$

$$(2_{270^\circ})^3 = (2^3)_{3 \times 270} = 8_{810^\circ} = 8_{90^\circ}$$

$$b) (2_{60^\circ})^4 = (2^4)_{4 \times 60} = 16_{240^\circ}$$

$$(2_{150^\circ})^4 = (2^4)_{4 \times 150} = 16_{600^\circ} = 16_{240^\circ}$$

$$(2_{270^\circ})^4 = (2^4)_{4 \times 270} = 16_{1080^\circ} = 16_{0^\circ}$$

$$(2_{330^\circ})^4 = (2^4)_{4 \times 330} = 16_{1320^\circ} = 16_{240^\circ}$$

Ej. 29, p 82

Expresa en forma polar los siguientes n^o complejos:

a) $-1 + i$

b) $\frac{-\sqrt{3} + i}{1 + i}$

c) $\frac{1}{-1 + \sqrt{3}i}$

a) $z = -1 + i$

$$r = |z| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$$

$$\alpha = \operatorname{arctg} \frac{1}{-1} = \operatorname{arctg}(-1) = \frac{3\pi}{4}$$

$$\left. \begin{array}{l} r = \sqrt{2} \\ \alpha = \frac{3\pi}{4} \end{array} \right\} z = \sqrt{2} \angle \frac{3\pi}{4}$$

b) $z = \frac{-\sqrt{3} + i}{1 + i}$

Lo hacemos tratando el numerador y el denominador por separado para evitar la complejidad algebraica.

$$z_1 = -\sqrt{3} + i$$

$$r_1 = |z_1| = \sqrt{(-\sqrt{3})^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$$

$$\alpha_1 = \operatorname{arctg}\left(\frac{1}{-\sqrt{3}}\right) = \frac{5\pi}{6}$$

$$\left. \begin{array}{l} r_1 = 2 \\ \alpha_1 = \frac{5\pi}{6} \end{array} \right\} 2 \angle \frac{5\pi}{6}$$

$$z_2 = 1 + i$$

$$r_2 = |z_2| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\alpha_2 = \operatorname{arctg} \frac{1}{1} = \operatorname{arctg} 1 = \frac{\pi}{4}$$

$$\left. \begin{array}{l} r_2 = \sqrt{2} \\ \alpha_2 = \frac{\pi}{4} \end{array} \right\} \sqrt{2} \angle \frac{\pi}{4}$$

luego:

$$z = \frac{-\sqrt{3} + i}{1 + i} = \frac{(2) \angle \frac{5\pi}{6}}{(\sqrt{2}) \angle \frac{\pi}{4}} = (\sqrt{2}) \angle \frac{5\pi}{6} - \frac{\pi}{4} = \sqrt{2} \angle \frac{7\pi}{12}$$

$$c) z = \frac{1}{-1 + \sqrt{3}i} = \frac{-1 - \sqrt{3}i}{(-1 + \sqrt{3}i)(-1 - \sqrt{3}i)} = \frac{-1 - \sqrt{3}i}{1 + 3} = \frac{-1 - \sqrt{3}i}{4} =$$

algebraicamente

$$= -\frac{1}{4} - \frac{\sqrt{3}}{4}i$$

$$r = |z| = \sqrt{\left(-\frac{1}{4}\right)^2 + \left(\frac{\sqrt{3}}{4}\right)^2} = \sqrt{\frac{1}{16} + \frac{3}{16}} = \sqrt{\frac{4}{16}} =$$

$$= \frac{\sqrt{4}}{\sqrt{16}} = \frac{2}{4} = \boxed{\frac{1}{2}}$$

$$\angle = \arctan \frac{\frac{\sqrt{3}}{4}}{\frac{1}{4}} = \arctan \sqrt{3} = 60^\circ = \frac{\pi}{3}$$

luego:

$$z = \left(\frac{1}{2} \right) \frac{\pi}{3}$$

Comprobación:

$$\begin{array}{c} z_1 \\ \frac{1}{-1 + \sqrt{3}i} \end{array}$$

$$z_1 = 1 = 1 + 0i$$

$$|z_1| = \sqrt{1^2 + 0^2} = \sqrt{1} = 1$$

$$\angle_1 = \arctan\left(\frac{0}{1}\right) = \arctan(0) = 0 = \pi$$

$$1_{0^\circ} = 1_{180^\circ} = \boxed{1\pi}$$

$$z_2 = -1 + \sqrt{3}i$$

$$|z_2| = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{1 + 3} = \sqrt{4} = 2$$

$$\angle_2 = \arctan \frac{\sqrt{3}}{-1} = \arctan(-\sqrt{3}) = 120^\circ = \frac{2\pi}{3}$$

$$z = \frac{1}{-1 + \sqrt{3}i} = \frac{1\pi}{2\frac{2\pi}{3}} = \left(\frac{1}{2} \right) \pi - \frac{2\pi}{3} = \boxed{\left(\frac{1}{2} \right) \frac{\pi}{3}} \text{ csgd}$$

3.- Dados los complejos $z = 5_{45^\circ}$, $w = 2_{15^\circ}$, $t = 4i$,
obten en forma polar:

a) $z \cdot t$

b) $\frac{z}{w^2}$

c) $\frac{z^3}{wt^2}$

d) $\frac{z \cdot w^3}{t}$

a) $z \cdot t = 5_{45^\circ} \cdot 4i = 5_{45^\circ} \cdot 4_{90^\circ} = \boxed{20_{135^\circ}}$

b) $\frac{z}{w^2} = \frac{5_{45^\circ}}{(2_{15^\circ})^2} = \frac{5_{45^\circ}}{4_{30^\circ}} = \boxed{\left(\frac{5}{4}\right)_{15^\circ}}$

c) $\frac{z^3}{wt^2} = \frac{(5_{45^\circ})^3}{2_{15^\circ} \cdot (4_{90^\circ})^2} = \frac{125_{135^\circ}}{2_{15^\circ} \cdot 16_{180^\circ}} = \frac{125_{135^\circ}}{32_{195^\circ}} =$
 $= \left(\frac{125}{32}\right)_{-60^\circ} = \boxed{\left(\frac{125}{32}\right)_{300^\circ}}$

d) $\frac{z \cdot w^3}{t} = \frac{5_{45^\circ} \cdot (2_{15^\circ})^3}{4_{90^\circ}} = \frac{5_{45^\circ} \cdot 8_{45^\circ}}{4_{90^\circ}} = \frac{40_{90^\circ}}{4_{90^\circ}} =$
 $= 10_{0^\circ} = \boxed{10}$

4.- Expresa $\cos 3\alpha$ y $\sin 3\alpha$ en f. de $\sin \alpha$ y $\cos \alpha$ utilizando la fórmula de Moivre. Ten en cuenta:

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

La fórmula de Moivre:

$$(\cos \alpha + i \sin \alpha)^n = (1_\alpha)^n = 1_{n\alpha} = (\cos n\alpha + i \sin n\alpha)$$

$$\begin{aligned} (1_\alpha)^3 &= 1(\cos \alpha + i \sin \alpha)^3 = 1(\cos 3\alpha + i \sin 3\alpha) \\ &= \cos^3 \alpha + 3i \cos^2 \alpha \sin \alpha + 3i^2 \cos \alpha \sin^2 \alpha + i^3 \sin^3 \alpha \\ &= \cos^3 \alpha + 3\cos^2 \alpha \sin \alpha i - 3\cos \alpha \sin^2 \alpha - \sin^3 \alpha i = \end{aligned}$$

(separamos lo q. no tiene i de lo q. tiene)

$$= (\cos^3 \alpha - 3\cos \alpha \sin^2 \alpha) + i(3\cos^2 \alpha \sin \alpha - \sin^3 \alpha)$$

$$(1_\alpha)^3 = 1_{3\alpha} = \cos 3\alpha + i \sin 3\alpha$$

Por tanto:

$$\cos 3\alpha = \cos^3 \alpha - 3\cos \alpha \sin^2 \alpha$$

$$\sin 3\alpha = 3\cos^2 \alpha \sin \alpha - \sin^3 \alpha$$

4 (bis).

Exp. resuelto n° 2, p 155

Calcula $\cos 2\alpha$ y $\sin 2\alpha$ mediante la f. de Moivre.

$$(\cos \alpha + i \sin \alpha)^n = (\cos n\alpha + i \sin n\alpha)$$

$$(\cos \alpha + i \sin \alpha)^2 = (\cos 2\alpha + i \sin 2\alpha)$$

$$\begin{aligned} (\cos \alpha + i \sin \alpha)^2 &= \cos^2 \alpha + 2i \cos \alpha \sin \alpha + i^2 \sin^2 \alpha = \\ &= \cos^2 \alpha + 2\cos \alpha \sin \alpha i - \sin^2 \alpha = \\ &= (\cos^2 \alpha - \sin^2 \alpha) + i(2\cos \alpha \sin \alpha) \end{aligned}$$

$\cos 2\alpha$ $\sin 2\alpha$

luego:

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\sin 2\alpha = 2\cos \alpha \sin \alpha$$

$$2, \neq 158$$

Halla el valor que debe tener x para que el número

$$\frac{1+3xi}{3-4i} \text{ sea:}$$

a) Un n° real

b) Un n° imaginario puro

$$\begin{aligned} \frac{1+3xi}{3-4i} &= \frac{(1+3xi)(3+4i)}{(3-4i)(3+4i)} = \frac{3+4i+9xi-12x}{9+16} = \\ &= \frac{(3-12x) + (4+9x)i}{25} = \frac{3-12x}{25} + \frac{(4+9x)i}{25} \end{aligned}$$

a) Para que sea un n° real, la parte imaginaria ha de ser igual a cero.

$$\frac{4+9x}{25} = 0 ; \quad 4+9x = 0 ; \quad 9x = -4 ; \quad \boxed{x = -\frac{4}{9}}$$

b) Para que sea imaginario puro, la parte real ha de ser cero.

$$\frac{3-12x}{25} = 0 ; \quad 3-12x = 0 ; \quad -12x = -3 ; \quad x = \frac{3}{12} = \boxed{\frac{1}{4}}$$

3, p 158

Resuelve la ecuación, en la que z es un n° complejo:

$$2z + 3i - 2 = \frac{3 + 2i - i}{2}$$

$$2(2z + 3i - 2) = 3 + 2i - i$$

$$4z + 6i - 4 = 3 + 2i - i$$

$$4z - 2i - 4 - 3 = -i - 6i$$

$$4z - 2i - 7 = -7i$$

$$z(4 - i) = 7 - 7i$$

$$z = \frac{7 - 7i}{4 - i} = \frac{(7 - 7i)(4 + i)}{(4 - i)(4 + i)} = \frac{28 + 7i - 28i + 7}{16 + 1} =$$

$$= \frac{35 - 21i}{17} = \boxed{\frac{35}{17} - \frac{21}{17}i}$$

7, p 160

Determina el valor de $\cos 15^\circ$ y $\sin 15^\circ$ a partir de la división $160^\circ : 145^\circ$

$$\frac{160^\circ}{145^\circ} = (1)_{60^\circ - 45^\circ} = 1_{15^\circ} = 1(\cos 15^\circ + i \sin 15^\circ) = \cos 15^\circ + i \sin 15^\circ$$

$$\frac{160^\circ}{145^\circ} = \frac{1(\cos 60^\circ + i \sin 60^\circ)}{1(\cos 45^\circ + i \sin 45^\circ)} = \frac{\frac{1}{2} + \frac{\sqrt{3}}{2}i}{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i} = \frac{1 + \sqrt{3}i}{\sqrt{2} + \sqrt{2}i} =$$

$$= \frac{(1 + \sqrt{3}i)(\sqrt{2} - \sqrt{2}i)}{(\sqrt{2} + \sqrt{2}i)(\sqrt{2} - \sqrt{2}i)} = \frac{\sqrt{2} - \sqrt{2}i + \sqrt{6}i + \sqrt{6}}{2 + 2} =$$

$$= \frac{\sqrt{2} + \sqrt{6} + (\sqrt{6} - \sqrt{2})i}{4} = \underbrace{\frac{\sqrt{2} + \sqrt{6}}{4}}_{\cos 15^\circ} + \underbrace{\frac{\sqrt{6} - \sqrt{2}}{4}i}_{\sin 15^\circ}$$

3.3.4 Ejercicios ^{resueltos} propuestos. Cálculo diferencial e integral. Prof. Javier Pérez.

780. (103 de 691)

[24] Calcular la parte real e imaginaria de $\frac{\bar{z}}{1+z^2}$ donde $z \in \mathbb{C} \setminus \{i, -i\}$.

$$z = x + iy, \quad x, y \in \mathbb{R}$$

$$\frac{\bar{z}}{1+z^2} = \frac{x-iy}{1+(x+iy)^2} = \frac{x-iy}{1+x^2+2xyi-y^2} = \frac{x-iy}{(1+x^2-y^2+2xyi)}$$

$$= \frac{(x-iy)(1+x^2-y^2-2xyi)}{(1+x^2-y^2+2xyi)(1+x^2-y^2-2xyi)} =$$

$$= \frac{x + x^3 - xy^2 - 2x^2yi - iy - iyx^2 + iy^3 - 2xy^2}{(1+x^2-y^2)^2 + 4x^2y^2} =$$

$$= \frac{x + x^3 - 3xy^2}{(1+x^2-y^2)^2 + 4x^2y^2} + \frac{-3x^2y - y + y^3}{(1+x^2-y^2)^2 + 4x^2y^2} i$$

$$\operatorname{Re} \left(\frac{\bar{z}}{1+z^2} \right)$$

$$\operatorname{Im} \left(\frac{\bar{z}}{1+z^2} \right)$$

25.- Calcular $\left| \frac{(2+i\sqrt{5})(1+i\sqrt{3})^3}{(\sqrt{5}+i\sqrt{3})} \right|$

Como lo que nos piden es el módulo no es preciso realizar las operaciones indicadas. Basta tener en cuenta que el módulo de un producto es el producto de los módulos, y, por tanto, el módulo de un cociente es el cociente de los módulos.

$$\left| \frac{(2+i\sqrt{5})(1+i\sqrt{3})^3}{(\sqrt{5}+i\sqrt{3})} \right| = \frac{|(2+i\sqrt{5})| |(1+i\sqrt{3})|^3}{|(\sqrt{5}+i\sqrt{3})|} =$$

$$|(2+i\sqrt{5})| = \sqrt{2^2 + (\sqrt{5})^2} = \sqrt{4+5} = \sqrt{9} = 3$$

$$|(1+i\sqrt{3})| = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

$$|(\sqrt{5}+i\sqrt{3})| = \sqrt{(\sqrt{5})^2 + (\sqrt{3})^2} = \sqrt{5+3} = \sqrt{8} = 2\sqrt{2}$$

$$= \frac{3 \times 8}{2\sqrt{2}} = \frac{12\sqrt{2}}{2} = \boxed{6\sqrt{2}}$$

26.- Calcular los n° complejos z , tales que $w = \frac{2z-i}{2+zi}$ es:

a) Un n° real

b) Un n° imaginario puro

Si $z = x+iy$, $x, y \in \mathbb{R}$

$$\begin{aligned} w &= \frac{2(x+iy)-i}{2+(x+iy)i} = \frac{2x+2yi-i}{2+xi-y} = \frac{2x+(2y-1)i}{(2-y+xi)} = \\ &= \frac{(2x+(2y-1)i)(2-y-xi)}{(2-y+xi)(2-y-xi)} = \frac{4x-2xy-2x^2i+(4y-2)i-(2y^2-y)i}{(2-y)^2+x^2} = \end{aligned}$$

$$\frac{4x - 2xy - 2x^2i + 4yi - 2i - 2y^2i + yi + 2xy - x}{(2-y)^2 + x^2}$$

a) Para que sea un n° real: $\text{Im}(w) = 0$

$$-2x^2 - 2y^2 + 5y - 2 = 0$$

$$x^2 + \left(y - \frac{5}{4}\right)^2 = \frac{9}{16}$$

z está en una circunferencia de centro $(0, \frac{5}{4})$ y radio $\frac{3}{4}$

b) para que seja imaginário puro: $\text{Re}(w) = 0$

$$3x = 0 \quad ; \quad \boxed{x = 0}$$

27.- Calcular los n° complejos z tales que $w = \frac{z-1-i}{z+1+i}$

- a) Es un n° real
b) tiene módulo 1

Si $z = x + iy$; $x, y \in \mathbb{R}$, tenemos:

$$\begin{aligned} w &= \frac{x+iy-1-i}{x+iy+1+i} = \frac{x-1+(y-1)i}{x+1+(y+1)i} = \\ &= \frac{[x-1+(y-1)i][x+1-(y+1)i]}{[x+1+(y+1)i][x+1-(y+1)i]} = \\ &= \frac{x^2+y^2-2+i(2y-2x)}{(x+1)^2+(y+1)^2} \end{aligned}$$

a) Si es un n° real: $\text{Im}(w) = 0$

$$2y - 2x = 0; \quad y = x \quad \rightarrow$$

Es decir está en la
bisector del 1^{er} y
3^{er} cuadrante.

$$b) |w| = 1 \Rightarrow |z-1-i| = |z+1+i|$$

$$\Rightarrow (x-1)^2 + (y-1)^2 = (x+1)^2 + (y+1)^2$$

$$\Rightarrow x + y = 0$$

Es decir z está en la bisector
del 2^o y 4^o cuadrante.

7, pag 162

Determina k para que el cociente $\frac{k+i}{1+i}$ sea igual a $2-i$

$$\begin{aligned}\frac{k+i}{1+i} &= \frac{(k+i)(1-i)}{(1+i)(1-i)} = \frac{k - ki + i + 1}{1+1} = \frac{(k+1) + (1-k)i}{2} = \\ &= \frac{k+1}{2} + \frac{(1-k)i}{2} = 2-i\end{aligned}$$

$$\frac{k+1}{2} = 2 ; \quad k+1 = 4 ; \quad \boxed{k=3}$$

$$\frac{1-k}{2} = -1 ; \quad 1-k = -2 ; \quad -k = -3 ; \quad \boxed{k=3}$$

8, pag 162

Calcular a y b de modo que se verifique:

$$(a+bi)^2 = 3+4i$$

$$(a+bi)^2 = a^2 + 2abi - b^2 = (a^2 - b^2) + 2abi$$

$$(a^2 - b^2) + 2abi = 3 + 4i$$

$$\left. \begin{aligned}a^2 - b^2 &= 3 \\ 2ab &= 4\end{aligned} \right\} \quad b = \frac{4}{2a}$$

$$a^2 - \left(\frac{4}{2a}\right)^2 = 3 ; \quad a^2 - \frac{16}{4a^2} = 3$$

$$4a^4 - 16 = 12a^2 ; \quad 4a^4 - 12a^2 - 16 = 0 ;$$

$$\boxed{a^4 - 3a^2 - 4 = 0}$$

Llamamos:

$$a^2 = x, \quad a^4 = x^2$$

Sustituyendo en la ecuación

$$x^2 - 3x - 4 = 0$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-4)}}{2} = \frac{3 \pm \sqrt{9+16}}{2} = \frac{3 \pm 5}{2} =$$

$$= \rightarrow \frac{8}{2} = \boxed{4}$$

$$\rightarrow \frac{-2}{2} = -1 \text{ (NO VALE)}$$

$$a = \pm \sqrt{x} = \pm \sqrt{4} = \boxed{\pm 2}$$

$$a = \boxed{2}; \quad b = \frac{4}{4} = \boxed{1}$$

$$a = \boxed{-2}; \quad b = \frac{4}{-4} = \boxed{-1}$$

p 162.

9.- Dados los complejos $2 - ai$ y $3 - bi$, halla a y b para que su producto sea igual a $8 + 4i$.

$$(2 - ai)(3 - bi) = 8 + 4i$$

$$6 - 2bi - 3ai - ab = 8 + 4i$$

$$(6 - ab) - (2b + 3a)i = 8 + 4i$$

$$\left. \begin{array}{l} 6 - ab = 8 \\ 2b + 3a = 4 \end{array} \right\} \begin{array}{l} -ab = 8 - 6 = 2 \\ 2(-\frac{2}{a}) + 3a = 4 \end{array}; \quad ab = -2; \quad b = -\frac{2}{a}$$

$$-4 + 3a^2 = 4a; \quad 3a^2 - 4a - 4 = 0$$

$$a = \frac{4 \pm \sqrt{16 + 48}}{6} = \frac{4 \pm \sqrt{64}}{6} = \frac{4 \pm 8}{6} \quad \begin{array}{l} \frac{12}{6} = \boxed{2}; \quad b = \boxed{-1} \\ -\frac{4}{6} = \boxed{-\frac{2}{3}}; \quad b = \boxed{3} \end{array}$$

162

10.- Calcular el valor de a y b para que se verifique:

$$a-3i = \frac{2+bi}{5-3i}$$

$$(a-3i)(5-3i) = 2+bi$$

$$5a-3ai-15i-9 = 2+bi$$

$$(5a+9) + (-3a-15)i = 2+bi$$

$$\left. \begin{array}{l} 5a+9=2 \\ -3a-15=b \end{array} \right\} \begin{array}{l} a = -\frac{7}{5} \\ b = -\frac{54}{5} \end{array}$$

11.- Hallar el valor de b para que el producto $(3-bi)(4+bi)$ sea:

a) Un n^o imaginario puro

b) " " real

$$\begin{aligned} a) (3-bi)(4+bi) &= 12 + 3bi - 24i + 6b = \\ &= (12+6b) + (3b-24)i \end{aligned}$$

La $PR=0$

$$12+6b=0 ; 6b=-12 ; b = -\frac{12}{6} = \boxed{-2}$$

b) la $PI=0$

$$3b-24=0 ; 3b=24 ; b = \frac{24}{3} = \boxed{8}$$

12.- Determina a para que $(a-2i)^2$ sea un n° imaginario puro.

$$(a-2i)^2 = a^2 - 4ai - 4 = (a^2 - 4) - 4ai$$

Para que sea imaginario puro; $PR=0$

$$a^2 - 4 = 0 ; a^2 = 4 ; a = \pm \sqrt{4} = \boxed{\pm 2}$$

13.- Calcula x para que el resultado del producto $(x+2+ix)(x-i)$ sea un n° real.

$$\begin{aligned}(x+2+ix)(x-i) &= x^2 - xi + 2x - 2i + x^2i + x = \\ &= (x^2 + 3x) + (-x - 2 + x^2)i\end{aligned}$$

Si ha de ser un n° real $PI=0$

$$x^2 - x - 2 = 0 ; x = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm \sqrt{9}}{2} = \frac{1 \pm 3}{2} =$$

$$\rightarrow \frac{4}{2} = \boxed{2}$$

$$\rightarrow -\frac{2}{2} = \boxed{-1}$$

p. 163

20.- Calcula m para que el n° complejo $z = 2\sqrt{5} + \sqrt{5}i$ sea un número real.

$$z = 2\sqrt{5} + \sqrt{5}i$$

$$|z| = \sqrt{(2\sqrt{5})^2 + (\sqrt{5})^2} = \sqrt{20+5} = \sqrt{25} = \boxed{5}$$

$$w = 3 - mi$$

$$|w| = \sqrt{3^2 + m^2} = \sqrt{9+m^2}$$

Como los módulos han de ser iguales:

$$\sqrt{9+m^2} = 5 ; 9+m^2 = 25 ; -m^2 = 16$$

$$m = \pm \sqrt{16} = \boxed{\pm 4}$$

4.M.1. Práctica 1

4.- Hallar dos n^{os} complejos tales que el primero sea imaginario puro, el módulo del segundo sea igual a $\frac{\sqrt{2}}{2}$ y que la suma de ambos n^{os} sea igual a su producto.

$$z_1 = iy_1$$

$$z_2 = x_2 + iy_2$$

CONDICIONES:

$$|z_2| = \sqrt{x_2^2 + y_2^2} = \frac{\sqrt{2}}{2}$$

$$iy_1 + x_2 + iy_2 = iy_1(x_2 + iy_2)$$

Planteamos el sistema siguiente:

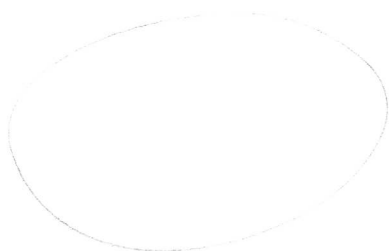
$$\left. \begin{array}{l} x_2^2 + y_2^2 = \frac{1}{2} \\ x_2 = -y_1 y_2 \\ y_1 + y_2 = x_2 y_1 \end{array} \right\} \begin{array}{l} z_1 = i \quad y \quad z_2 = \frac{1}{2} - \frac{1}{2}i \\ z_1 = -i \quad y \quad z_2 = \frac{1}{2} + \frac{1}{2}i \end{array}$$

Desarrollo:

$$iy_1 + x_2 + iy_2 = iy_1 x_2 - y_1 y_2$$

$$x_2 + y_1 y_2 = iy_1 x_2 - iy_1 - iy_2 = i(y_1 x_2 - y_1 - y_2)$$

¡ No veo lo que sale !



21.- La presa en forma polar z , su opuesto $-z$ y su conjugado $\bar{z}$ en cada uno de estos casos:

a) $z = 1 - \sqrt{3}i$; b) $z = -2 - 2i$; c) $z = -2\sqrt{3} + 2i$

	z	$-z$	$\bar{z}$
a)	$1 - \sqrt{3}i$	$-1 + \sqrt{3}i$	$1 + \sqrt{3}i$
b)	$-2 - 2i$	$2 + 2i$	$-2 + 2i$
c)	$-2\sqrt{3} + 2i$	$2\sqrt{3} - 2i$	$-2\sqrt{3} - 2i$

Hacerlo con MATHEMATICA

4, p149

Sabemos que $i^2 = -1$, calcula:

$$\left. \begin{aligned} i^0 &= 1 \\ i^1 &= i \\ i^2 &= -1 \\ i^3 &= -i \end{aligned} \right\}$$

GRUPO QUE SE REPITE

$$\boxed{i^4 = 1}$$

$$i^5 = i$$

$$i^6 = -1$$

⋮

$$i^{20} = i^0 = 1$$

$$i^{21} = i^1 = i$$

$$i^{22} = i^2 = -1$$

$$i^{23} = i^3 = -i$$

$$\begin{aligned} i^n &= i^{4c+r} = i^{4c} \times i^r = (i^4)^c \times i^r = \\ &= 1^c \times i^r = 1 \cdot i^r = i^r \end{aligned}$$

$$\boxed{i^n = i^r}$$

20	4	22	4
0	5	2	5
21	4	23	4
1	5	3	5
	n		4
	c		c

1, pag 151

a) $(6-5i) + (2-i) - 2(-5+6i) =$

$$= 6-5i+2-i+10-12i = \boxed{18-18i}$$

b) $(2-3i) - (5+4i) + \frac{1}{2}(6-4i) =$

$$= \cancel{2}-3i-\cancel{5}-4i+\cancel{3}-2i = \boxed{-9i}$$

c) $(3+2i)(4-2i) = 12-6i+8i-4i^2 =$

$$= 12-6i+8i+4 = \boxed{16+2i}$$

d) $(2+3i)(5-6i) = 10-12i+15i-18i^2 =$

$$= 10-12i+15i+18 = \boxed{28+3i}$$

$$\begin{aligned}
 e) \quad (-i+1)(3-2i)(1+3i) &= (-3i+2i^2+3-2i)(1+3i) = \\
 &= (-3i-2+3-2i)(1+3i) = (1-5i)(1+3i) = \\
 &= 1+3i-5i-15i^2 = 1+3i-5i+15 = \boxed{16-2i}
 \end{aligned}$$

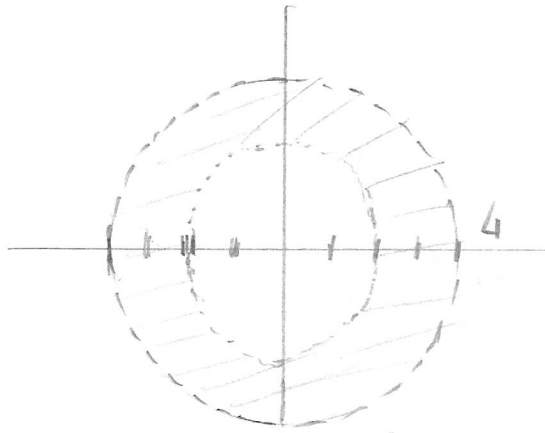
$$\begin{aligned}
 f) \quad \frac{2+4i}{4-2i} &= \frac{(2+4i)(4+2i)}{(4-2i)(4+2i)} = \frac{8+4i+16i+8i^2}{16+4} = \frac{8+4i+16i-8}{20} = \\
 &= \frac{20i}{20} = \boxed{i}
 \end{aligned}$$

$$\begin{aligned}
 g) \quad \frac{1-4i}{3+i} &= \frac{(1-4i)(3-i)}{(3+i)(3-i)} = \frac{3-i-12i+4i^2}{9+1} = \\
 &= \frac{3-i-12i-4}{10} = \frac{-1-13i}{10} = \boxed{\frac{1}{10} - \frac{13i}{10}}
 \end{aligned}$$

$$\begin{aligned}
 k) \quad \frac{4-2i}{i} &= \frac{(4-2i)(-i)}{i(-i)} = \frac{-4i+2i^2}{-i^2} = \frac{-4i-2}{1} = \\
 &= \boxed{-2-4i}
 \end{aligned}$$

$$l) \quad 6-3\left(5+\frac{2}{5}i\right) = 6-15-\frac{6}{5}i = \boxed{-9-\frac{6}{5}i}$$

$$\begin{aligned}
 m) \quad \frac{(-3i)^2(1-2i)}{2+2i} &= \frac{-9(1-2i)}{2+2i} = \frac{-9+18i}{2+2i} = \\
 &= \frac{(-9+18i)(2-2i)}{(2+2i)(2-2i)} = \frac{-18+18i+36i-36i^2}{4+4} = \\
 &= \frac{-18+18i+36i+36}{8} = \frac{18+54i}{8} = \frac{18}{8} + \frac{54i}{8} = \boxed{\frac{9}{4} + \frac{27}{4}i}
 \end{aligned}$$



p162

4.- Calcular:

a) i^{37} b) i^{256} c) i^{87} d) i^{64} e) i^{-216}

a) $i^{37} = i^1 = \boxed{i}$

b) $i^{256} = i^0 = \boxed{1}$

c) $i^{87} = i^3 = \boxed{-i}$

d) $i^{64} = i^0 = \boxed{1}$

e) $i^{-256} = \frac{1}{i^{256}} = \frac{1}{i^0} = \frac{1}{1} = \boxed{1}$

$$\begin{array}{r} 37 \overline{) 4} \\ \underline{36} \\ 1 \\ \underline{0} \\ 0 \\ \underline{0} \\ 0 \end{array}$$

$$\begin{array}{r} 256 \overline{) 4} \\ \underline{256} \\ 0 \\ \underline{0} \\ 0 \\ \underline{0} \\ 0 \end{array}$$

$$\begin{array}{r} 87 \overline{) 4} \\ \underline{84} \\ 3 \\ \underline{3} \\ 0 \\ \underline{0} \\ 0 \end{array}$$

$$\begin{array}{r} 64 \overline{) 4} \\ \underline{64} \\ 0 \\ \underline{0} \\ 0 \\ \underline{0} \\ 0 \end{array}$$

$$\begin{array}{l} i^0 = 1 \\ i^1 = i \\ i^2 = -1 \\ i^3 = -i \end{array}$$

5.- Dado el n° complejo $z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$, prueba que:

a) $1 + z + z^2 = 0$

b) $\frac{1}{z} = z^2$

$$z^2 = \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^2 = \left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}i\right) + \left(\frac{\sqrt{3}}{2}i\right)^2 =$$
$$= \frac{1}{4} - \frac{\sqrt{3}}{2}i - \frac{3}{4} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

a) $\cancel{1} + \left(\cancel{-\frac{1}{2} + \frac{\sqrt{3}}{2}i}\right) + \left(\cancel{-\frac{1}{2} - \frac{\sqrt{3}}{2}i}\right) = 0$; $\boxed{0=0}$

b) $\frac{1}{z} = \frac{1}{-\frac{1}{2} + \frac{\sqrt{3}}{2}i} = \frac{1}{\frac{-1 + \sqrt{3}i}{2}} = \frac{2}{-1 + \sqrt{3}i} =$

$$= \frac{2(-1 - \sqrt{3}i)}{(-1 + \sqrt{3}i)(-1 - \sqrt{3}i)} = \frac{2(-1 - \sqrt{3}i)}{1 + 3} =$$
$$= \frac{2(\quad)}{4} = \frac{-1 - \sqrt{3}i}{2} = \boxed{-\frac{1}{2} - \frac{\sqrt{3}}{2}i}$$

$\underline{\underline{z^2}}$

6.- Calcula m y n para que se verifique la igualdad

$$(2 + mi) + (n + 5i) = 7 - 2i$$

$$2 + mi + n + 5i = 7 - 2i$$

$$(2 + n) + (5 + m)i = 7 - 2i$$

$$2 + n = 7 \quad ; \quad n = 7 - 2 = \boxed{5}$$

$$5 + m = -2 \quad ; \quad m = -2 - 5 = \boxed{-7}$$

1. M. 1, Práctica 1

1.- Halla el módulo y el argumento de $(1+i\sqrt{3})^{18}$

pasamos el complejo a polar:

$$\left. \begin{aligned} r &= \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2 \\ \alpha &= \arctg \frac{\sqrt{3}}{1} = \arctg \sqrt{3} = 60^\circ = \frac{\pi}{3} \end{aligned} \right\} r_\alpha = 2 \frac{\pi}{3}$$

luego:

$$(1+i\sqrt{3})^{18} = \left(2 \frac{\pi}{3} \right)^{18} = \left(2 \frac{\pi}{3} \right)^{18} \frac{\pi}{3} = 2^{18} 6\pi$$

El módulo es: 2^{18}

El argumento: 6π

RADICACIÓN DE NÚM COMPLEJOS

Ej. p 159. ANAYA. 1º Bachillerato

Calcular:

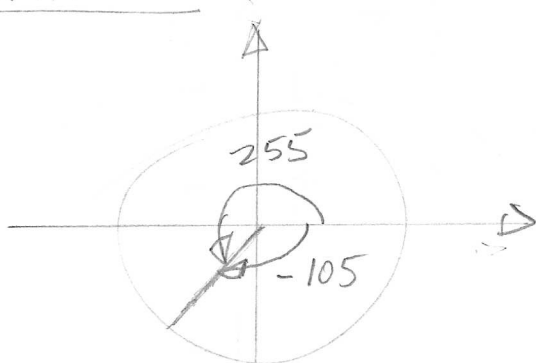
$$z_1 = \sqrt{3} + i \quad \left. \begin{array}{l} \rightarrow r_1 = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = \boxed{2} \\ \rightarrow \alpha_1 = \arctg \frac{1}{\sqrt{3}} = \boxed{30^\circ} \end{array} \right\} 2_{30^\circ}$$

$$z_2 = -1 + i \quad \left. \begin{array}{l} \rightarrow r_2 = \sqrt{(-1)^2 + 1^2} = \boxed{\sqrt{2}} \\ \rightarrow \alpha_2 = \arctg \left(\frac{1}{-1} \right) = \arctg(-1) = \boxed{135^\circ} \end{array} \right\} \sqrt{2}_{135^\circ}$$

luego:

$$\sqrt[3]{\left(\frac{\sqrt{3}+i}{-1+i} \right)^2} = \sqrt[3]{\left[\frac{2_{30}}{\sqrt{2}_{135}} \right]^2} = \sqrt[3]{\left[\frac{2}{\sqrt{2}} \right]_{-105^\circ}^2} = \sqrt[3]{\left[\frac{2}{\sqrt{2}} \right]_{255}^2}$$

INTERPRETACIÓN



$$= \sqrt[3]{\left[\frac{2}{\sqrt{2}} \right]_{255}^2} = \sqrt[3]{\sqrt{2}_{255}} = \sqrt[3]{2_{510}} = \sqrt[3]{2_{150^\circ}}$$

RACIONALIZADO

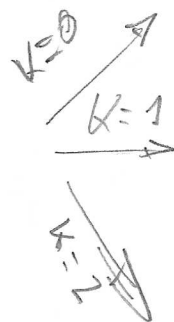
< 360°

luego:

$$\sqrt[n]{R\beta} \quad \begin{matrix} \nearrow \\ \searrow \end{matrix} \quad \begin{matrix} r = \sqrt[n]{R} \\ \alpha = \frac{\beta + 360k}{n} \end{matrix}$$

$$r = \sqrt[3]{2}$$

$$\alpha = \frac{150^\circ + 360k}{3} = 50 + 120k$$



$\sqrt[3]{2}$	50°
$\sqrt[3]{2}$	170°
$\sqrt[3]{2}$	290°

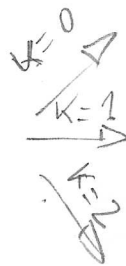
Ej. p160. ANAYA, 1º Bachillerato

$$\sqrt[3]{-64} = \sqrt[3]{-64 + 0i} = \sqrt[3]{64, 180^\circ}$$

$$\sqrt[n]{R\beta} \quad \begin{matrix} \nearrow \\ \searrow \end{matrix} \quad \begin{matrix} r = \sqrt[n]{R} \\ \alpha = \frac{\beta + 360k}{n} \end{matrix}$$

$$r = \sqrt[3]{64} = 4$$

$$\alpha = \frac{180 + 360k}{3} = 60 + 120k$$



4	60°
4	180°
4	300°

En forma binómica:

$$4_{60^\circ} = 4 (\cos 60 + i \sin 60) = 2 + 2\sqrt{3}i$$

$$4_{180^\circ} = 4 (\cos 180 + i \sin 180) = -4$$

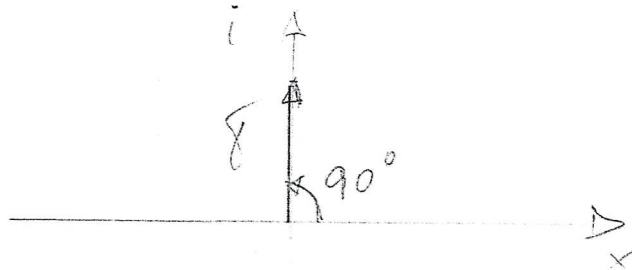
$$4_{300^\circ} = 4 (\cos 300 + i \sin 300) = 2 - 2\sqrt{3}i$$

El resultado, $\neq 157$. ANAYA. 1º bache.

$$\sqrt[3]{8i}$$

Representamos el n.º complejo en polar:

$$8i = 890^\circ$$

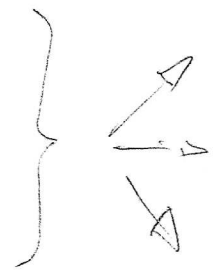


Su módulo:

$$|z| = \sqrt[n]{R} = \sqrt[3]{8} = \boxed{2}$$

Su argumento:

$$\alpha = \frac{\beta + 360k}{n} = \frac{90 + 360k}{3} = \boxed{30 + 120k}$$

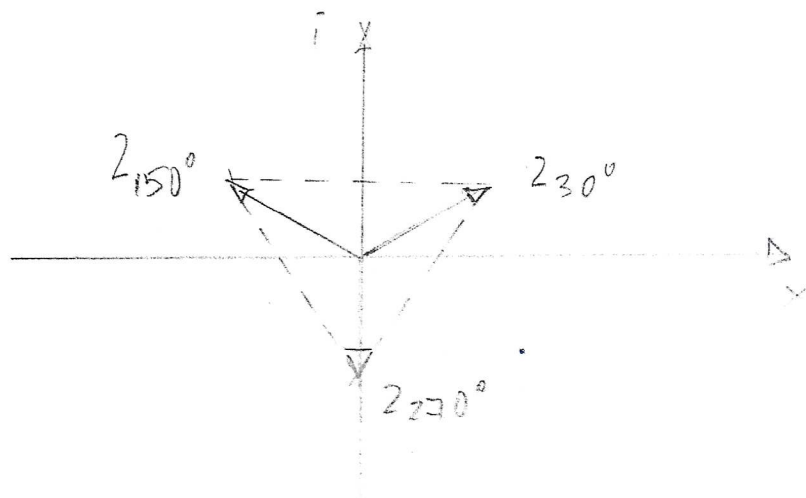


$$k=0, \alpha_1 = 30^\circ$$

$$k=1, \alpha_2 = 150^\circ$$

$$k=2, \alpha_3 = 270^\circ$$

Y representamos sus raíces:



1. resuelto 2, p 157. ANAYA. 1º BACHILLER

$$\sqrt[4]{-\frac{1}{2} - \frac{\sqrt{3}}{2}i}$$

El módulo de R:

$$|R| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = \boxed{1}$$

por tanto:

$$r = \sqrt[4]{R} = \sqrt[4]{1} = \boxed{1}$$

El argumento:

$$\beta = \operatorname{arctg} \frac{+\sqrt{3}}{+\frac{1}{2}} = \operatorname{arctg} \sqrt{3} = \boxed{240^\circ}$$

Luego:

$$\alpha = \frac{\beta + 360k}{n} = \frac{240 + 360k}{4} = \underline{\underline{60 + 90k}}$$

Las raíces son:

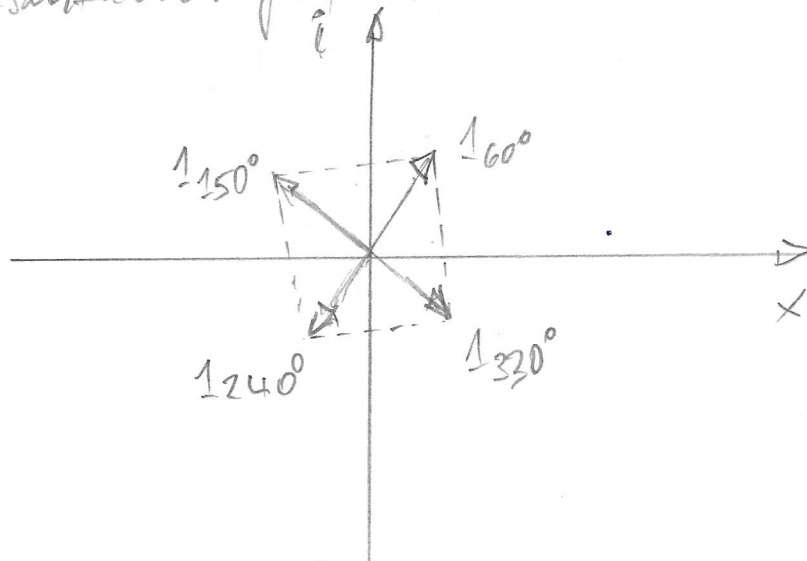
$$k=0, \quad 160^\circ$$

$$k=1, \quad 1150^\circ$$

$$k=2, \quad 1240^\circ$$

$$k=3, \quad 1330^\circ$$

Se representación gráfica:



PROBLEMAS PROPUESTOS. F.M.I. TEMA 01. COMPLEJOS

1.- Calcular $\frac{z_1 \cdot z_2 - z_3 \cdot z_4}{z_1 - z_2}$

Simulo:

$$z_1 = 1 + i$$

$$z_2 = 1 - i$$

$$z_3 = i$$

$$z_4 = -i$$

$$\frac{z_1 \cdot z_2 - z_3 \cdot z_4}{z_1 - z_2} = \frac{(1+i)(1-i) - i(-i)}{1+i - (1-i)} = \frac{\cancel{1} + \cancel{i^2} + 1}{\cancel{1} + i - \cancel{1} + i} = \frac{1}{2i}$$

RACIONALIZANDO:

$$\frac{1}{2i} \cdot \frac{-2i}{-2i} = \frac{-2i}{-2i \cdot 2i} = \frac{-2i}{-4i^2} = \frac{-2i}{-4(-1)} = \frac{-2i}{4} = \boxed{-\frac{i}{2}}$$

2.- Calcular $\left| \frac{z_1 + 2z_2 - 5 - i}{2z_1 - z_2 + 3 - i} \right|^2$

Simulo:

$$z_1 = 2 + i$$

$$z_2 = 3 - 2i$$

$$\left| \frac{z_1 + 2z_2 - 5 - i}{2z_1 - z_2 + 3 - i} \right|^2 = \left| \frac{2+i + 2(3-2i) - 5 - i}{2(2+i) - (3-2i) + 3 - i} \right|^2 =$$

$$= \left| \frac{\cancel{2} + \cancel{i} + 6 - 4i - 5 - \cancel{i}}{4 + 2i - 3 + 2i + 3 - i} \right|^2 = \left| \frac{3 - 4i}{4 + 3i} \right|^2 =$$

$$= \left| \frac{(3-4i)(4-3i)}{(4+3i)(4-3i)} \right|^2 = \left| \frac{\cancel{12} - 9i - 16i - \cancel{12}}{16 - 9} \right|^2 =$$

$$= \left| \frac{-25i}{7} \right|^2 = \left| -\frac{25}{7}i \right|^2 = \left(\frac{25}{7} \right)^2 \cdot i^2 = \frac{625}{49} \cdot (-1) = \boxed{-\frac{625}{49}}$$

5.- Realizar las siguientes operaciones:

a) $\sqrt[4]{-16}$, $\sqrt[3]{i}$, $\sqrt{1+i}$

$\sqrt[4]{-16}$

El complejo en polar:

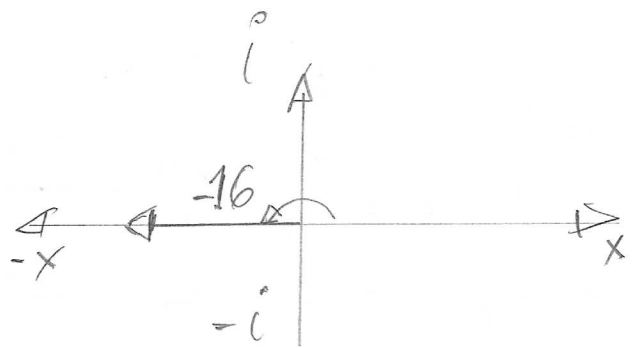
$$z = (-16) + i0$$

Su módulo:

$$|z| = \sqrt{(-16)^2} = 16$$

Su argumento:

$$\beta = \arctg \frac{0}{-16} = \arctg(0) = \pi$$



$$r = \sqrt[4]{16} = 2$$

$$\alpha = \frac{\beta + 2k\pi}{4} = \frac{\pi + 2k\pi}{4} = \frac{\pi}{4} + \frac{k\pi}{2}$$

$$k=0, \alpha = \frac{\pi}{4}$$

$$k=1, \alpha = \frac{\pi}{4} + \frac{\pi}{2} = \frac{\pi + 2\pi}{4} = \frac{3\pi}{4}$$

$$k=2, \alpha = \frac{\pi}{4} + \pi = \frac{5\pi}{4}$$

$$k=3, \alpha = \frac{\pi}{4} + \frac{3\pi}{2} = \frac{\pi + 6\pi}{4} = \frac{7\pi}{4}$$

$\sqrt[3]{i}$

El complejo en polar:

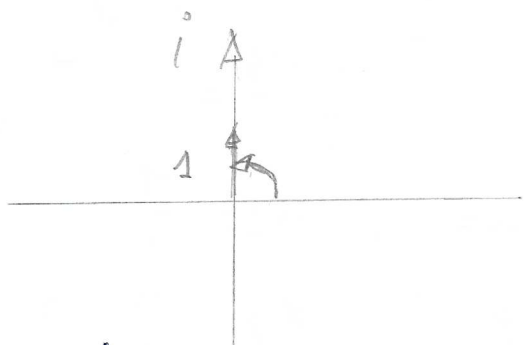
$$z = (1) + i$$

Su módulo:

$$|z| = \sqrt{0+1^2} = \sqrt{1} = 1$$

Su argumento:

$$\alpha = \arctg\left(\frac{1}{0}\right) = \arctg(\infty) = \frac{\pi}{2}$$



φ 157

1.- Halla las raíces sextas de 1. Representálas y expresasas en f. binómica.

$$\sqrt[n]{R\beta} \quad \begin{matrix} \nearrow r = \sqrt[n]{R} \\ \searrow \alpha = \frac{\beta + 2k\pi}{n} \end{matrix}$$

$$\sqrt[6]{1} = \sqrt[6]{10^\circ}$$

$$r = \sqrt[6]{1} = 1$$

$$\beta = 0$$

Luego:

$$\alpha = \frac{0 + 2k\pi}{6}$$

$$k=0; \alpha_1 = \frac{0}{6} = 0$$

$$k=1; \alpha_2 = \frac{2\pi}{6} = \frac{\pi}{3}$$

$$k=2; \alpha_3 = \frac{4\pi}{6} = \frac{2\pi}{3}$$

$$k=3; \alpha_4 = \frac{6\pi}{6} = \pi$$

$$k=4; \alpha_5 = \frac{8\pi}{6} = \frac{4\pi}{3}$$

$$k=5; \alpha_6 = \frac{10\pi}{6} = \frac{5\pi}{3}$$

f. POLAR	f. BINOMICA
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$$1_0^\circ = 1 + 0i$$

$$1_{\frac{\pi}{3}} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$1_{\frac{2\pi}{3}} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$1_\pi = -1 + 0i$$

$$1_{\frac{4\pi}{3}} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$1_{\frac{5\pi}{3}} = \frac{1}{2} - \frac{\sqrt{3}}{2}i$$

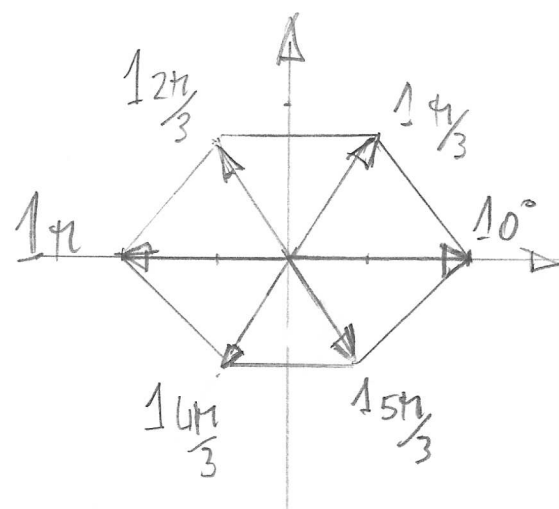
CÁLCULOS

$$1_{\frac{\pi}{3}} = 1 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) = \left(\frac{1}{2} + i \frac{\sqrt{3}}{2} \right)$$

$$1_{\frac{4\pi}{3}} = 1 \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right) = \left(-\frac{1}{2} - i \frac{\sqrt{3}}{2} \right)$$

$$1_{\frac{2\pi}{3}} = 1 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right) = \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i \right)$$

$$1_{\frac{5\pi}{3}} = 1 \left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right) = \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i \right)$$



2.- Resuelve la ecuación $z^3 + 27 = 0$. Representa sus soluciones.

$$z^3 + 27 = 0 \quad ; \quad z^3 = -27 \quad ; \quad z = \sqrt[3]{-27}$$

$$\sqrt[3]{-27} = \sqrt[3]{27} \cdot i$$

$$r = \sqrt[3]{27} = \boxed{3}$$

$$\beta = \pi$$

$$\alpha = \frac{\beta + 2k\pi}{n} = \frac{\pi + 2k\pi}{3}$$

$$k=0 \quad ; \quad \alpha_1 = \frac{\pi}{3}$$

$$k=1 \quad ; \quad \alpha_2 = \frac{\pi + 2\pi}{3} = \frac{3\pi}{3} = \pi$$

$$k=2 \quad ; \quad \alpha_3 = \frac{\pi + 4\pi}{3} = \frac{5\pi}{3}$$

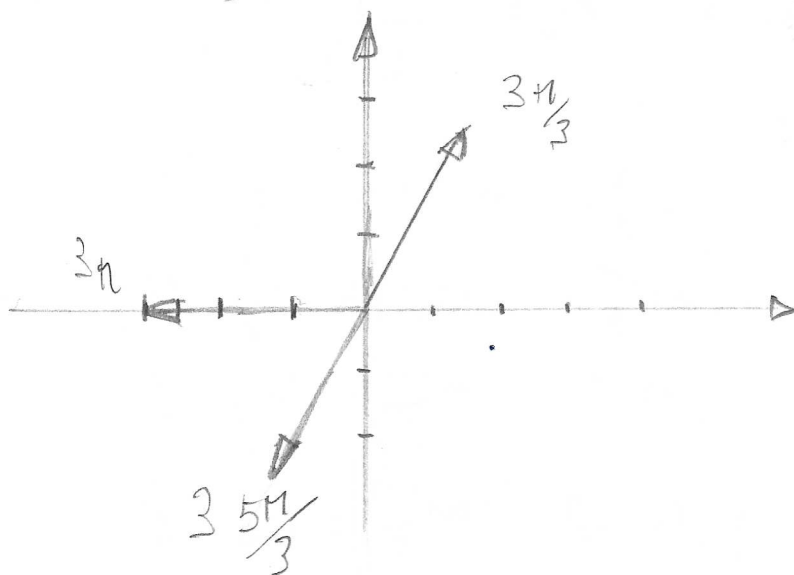
±. POLAR

±. BINOMICA

$$3 \frac{\pi}{3} = 3 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) = 3 \left(\frac{1}{2} + i \frac{\sqrt{3}}{2} \right) = \frac{3}{2} + \frac{3\sqrt{3}}{2}i$$

$$3 \pi = 3 \left(\cos \pi + i \sin \pi \right) = 3(-1 + 0i) = -3$$

$$3 \frac{5\pi}{3} = 3 \left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right) = 3 \left(\frac{1}{2} - i \frac{\sqrt{3}}{2} \right) = \frac{3}{2} - \frac{3\sqrt{3}}{2}i$$



3- Calcula

a) $\sqrt[3]{-i}$

b) $\sqrt[4]{-8+8\sqrt{3}i}$

c) $\sqrt{-25}$

d) $\sqrt[3]{\frac{-2+2i}{1+\sqrt{3}i}}$

e) $\sqrt[3]{-1}$

a) $\sqrt[3]{-i} = \sqrt[3]{1_{270^\circ}} = \sqrt[3]{1_{\frac{3\pi}{2}}}$

$\rho = \sqrt[3]{1} = 1$

$\beta = \frac{3\pi}{2}$

$\alpha = \frac{\beta + 2k\pi}{n} = \frac{\frac{3\pi}{2} + 2k\pi}{3}$

$k=0 ; \alpha_1 = \frac{3\pi}{6} = \frac{\pi}{2} \rightarrow 1_{\pi/2}$

$k=1 ; \alpha_2 = \frac{\frac{3\pi}{2} + 2\pi}{3} = \frac{7\pi}{6} \rightarrow 1_{7\pi/6}$

$k=2 ; \alpha_3 = \frac{\frac{3\pi}{2} + 4\pi}{3} = \frac{11\pi}{6} \rightarrow 1_{11\pi/6}$

$1_{\pi/2} = 1 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right) = 1(0 + i1) = i$

$1_{7\pi/6} = 1 \left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} \right) = 1 \left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i \right) = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$

$1_{11\pi/6} = 1 \left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right) = 1 \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) = \frac{\sqrt{3}}{2} + \frac{1}{2}i$

$$b) \sqrt[4]{-8+8\sqrt{3}i} =$$

pasamos el radicando a forma polar:

$$z = -8 + 8\sqrt{3}i$$

$$|z| = \sqrt{(-8)^2 + (8\sqrt{3})^2} = \sqrt{64 + 192} = \sqrt{256} = 16$$

$$\alpha = \arctg \frac{8\sqrt{3}}{-8} = \arctg(-\sqrt{3}) = -60^\circ = 120^\circ = \frac{2\pi}{3}$$

$$\sqrt[4]{-8+8\sqrt{3}i} = \sqrt[4]{16 e^{i\frac{2\pi}{3}}}$$

luego:

$$r = \sqrt[4]{16} = 2$$

$$\beta = \frac{2\pi}{3}$$

$$\alpha = \frac{\beta + 2k\pi}{4} = \frac{\frac{2\pi}{3} + 2k\pi}{4}$$

$$k=0; \alpha_1 = \frac{\frac{2\pi}{3}}{4} = \frac{2\pi}{12} = \left(\frac{\pi}{6}\right) \rightarrow 2 e^{i\frac{\pi}{6}}$$

$$k=1; \alpha_2 = \frac{\frac{2\pi}{3} + 2\pi}{4} = \frac{\frac{8\pi}{3}}{4} = \frac{8\pi}{12} = \left(\frac{4\pi}{3}\right) \rightarrow 2 e^{i\frac{4\pi}{3}}$$

$$k=2; \alpha_3 = \frac{\frac{2\pi}{3} + 4\pi}{4} = \frac{14\pi}{12} = \left(\frac{7\pi}{6}\right) \rightarrow 2 e^{i\frac{7\pi}{6}}$$

$$k=3; \alpha_4 = \frac{\frac{2\pi}{3} + 6\pi}{4} = \frac{20\pi}{12} = \left(\frac{5\pi}{3}\right) \rightarrow 2 e^{i\frac{5\pi}{3}}$$

$$2 e^{i\frac{\pi}{6}} = 2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) = 2 \left(\frac{\sqrt{3}}{2} + i \frac{1}{2} \right) = \sqrt{3} + i$$

$$2 e^{i\frac{4\pi}{3}} = 2 \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right) = 2 \left(-\frac{1}{2} + i \frac{\sqrt{3}}{2} \right) = -1 + \sqrt{3}i$$

$$2 e^{i\frac{7\pi}{6}} = 2 \left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} \right) = 2 \left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i \right) = -\sqrt{3} - i$$

$$2 e^{i\frac{5\pi}{3}} = 2 \left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right) = 2 \left(\frac{1}{2} - i \frac{\sqrt{3}}{2} \right) = 1 - \sqrt{3}i$$

$$c) \sqrt{-25} = \sqrt{25}i$$

$$r = \sqrt{25} = 5$$

$$\beta = \pi$$

$$\alpha = \frac{\beta + 2k\pi}{n} = \frac{\pi + 2k\pi}{2}$$

$$k=0; \alpha_1 = \frac{\pi}{2} \rightarrow 5^{1/2}$$

$$k=1; \alpha_2 = \frac{\pi + 2\pi}{2} = \frac{3\pi}{2} \rightarrow 5^{3/2}$$

$$5^{1/2} = 5 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right) = 5 (0 + i1) = 5i$$

$$5^{3/2} = 5 \left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right) = 5 (0 - i1) = -5i$$

$$d) \sqrt[3]{\frac{-2+2i}{1+\sqrt{3}i}}$$

Operamos el radicando:

$$\begin{aligned} \frac{-2+2i}{1+\sqrt{3}i} &= \frac{(-2+2i)(1-\sqrt{3}i)}{(1+\sqrt{3}i)(1-\sqrt{3}i)} = \frac{-2+2\sqrt{3}i+2i+2\sqrt{3}}{1+3} \\ &= \frac{(-2+2\sqrt{3}) + (2+2\sqrt{3})i}{4} = \frac{1+3}{2} \frac{(-1+\sqrt{3}) + (1+\sqrt{3})i}{2} \end{aligned}$$

luego pasamos a polar cada n° complejo. ~~SE~~ COMPLICAR...

$$z_1 = -2+2i$$

$$|z_1| = \sqrt{(-2)^2 + 2^2} = \sqrt{8}$$

$$\alpha_1 = \operatorname{arctg} \frac{2}{-2} = \operatorname{arctg} (-1) = -45^\circ = 135^\circ = \frac{3\pi}{4}$$

$$z_2 = 1+\sqrt{3}i$$

$$|z_2| = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

$$\alpha_2 = \operatorname{arctg} \frac{\sqrt{3}}{1} = \operatorname{arctg} \sqrt{3} = 60^\circ = \frac{\pi}{3}$$

$$\sqrt[3]{\frac{-2+2i}{1+\sqrt{3}i}} = \sqrt[3]{\frac{\sqrt{2} e^{i\pi/4}}{2 e^{i\pi/3}}} = \sqrt[3]{\sqrt{2} e^{i\pi/12}}$$

$$\frac{\sqrt{2} e^{i\pi/4}}{2 e^{i\pi/3}} = \left(\frac{\sqrt{2}}{2}\right) e^{i\pi/4 - i\pi/3} = \left(\frac{\sqrt{2}}{2}\right) e^{i\pi/12} = \sqrt{2} e^{i\pi/12}$$

75°

$$r = \sqrt[3]{\sqrt{2}} = \sqrt{2}$$

$$\beta = \frac{5\pi}{12}$$

$$\alpha = \frac{\beta + 2k\pi}{n} = \frac{\frac{5\pi}{12} + 2k\pi}{3}$$

$$k=0; \alpha_1 = \frac{5\pi}{36} = 25^\circ \rightarrow \sqrt[6]{2} 25^\circ$$

$$k=1; \alpha_2 = \frac{\frac{5\pi}{12} + 2\pi}{3} = \frac{29\pi}{36} = 145^\circ \rightarrow \sqrt[6]{2} 145^\circ$$

$$k=2; \alpha_3 = \frac{\frac{5\pi}{12} + 4\pi}{3} = \frac{53\pi}{36} = 265^\circ \rightarrow \sqrt[6]{2} 265^\circ$$

e) $\sqrt[3]{-1}$

$$\sqrt[3]{-1} = -1 \Rightarrow (-1)^3 = -1$$

REAL

$$\sqrt[3]{-1} = \cos \frac{\pi}{3} + i \sin \frac{\pi}{3}$$

COMPLEXO

$$\sqrt[3]{-1} = \sqrt[3]{-1+0i} = \sqrt[3]{1\pi}$$

F. BINOM. F. POLAR

$$r = \sqrt[3]{1} = 1$$

$$\alpha = \frac{\beta + 2k\pi}{n} = \frac{\pi + 2k\pi}{3}$$

$$k=0, \alpha_1 = \frac{\pi + 0}{3} = \frac{\pi}{3} \rightarrow 1\pi$$

$$k=1, \alpha_2 = \frac{\pi + 2\pi}{3} = \frac{3\pi}{3} = \pi \rightarrow 1\pi$$

$$k=2, \alpha_3 = \frac{\pi + 4\pi}{3} = \frac{5\pi}{3} \rightarrow 1\frac{5\pi}{3}$$

COMPARATIVA

6- El n° $4+3i$ es la raíz cuarta de un número complejo, Z .
Hallar las otras tres raíces cuartas de Z .

$$\sqrt[4]{Z} = 4+3i$$

El módulo:

$$r = \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$$

El argumento:

$$\alpha = \arctg \frac{3}{4} = 36,869^\circ = 36^\circ 52'$$

$$5_{36^\circ 52'}$$

Las otras tres raíces:

$$5_{36^\circ 52'} + 90^\circ = 5_{126^\circ 52'} = -3 + 4i$$

$$5_{36^\circ 52'} + 180^\circ = 5_{216^\circ 52'} = -4 - 3i$$

$$5_{36^\circ 52'} + 270^\circ = 5_{306^\circ 52'} = 3 - 4i$$

7.- Calcular las siguientes raíces y representar gráficamente sus soluciones:

a) $\sqrt{-9}$

b) $\sqrt[3]{-27}$

c) $\sqrt[3]{2-2i}$

d) $\sqrt[3]{\frac{1-i}{1+i}}$

e) $\sqrt[5]{-\frac{32}{i}}$

f) $\sqrt[3]{8i}$

$$a) \sqrt{-9} = \sqrt{9}i$$

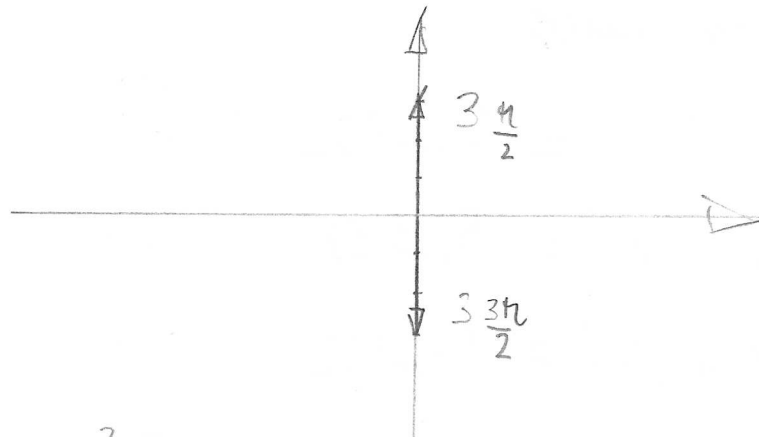
$$r = \sqrt[2]{9} = 3$$

$$\beta = \pi$$

$$\alpha = \frac{\beta + 2k\pi}{2} = \frac{\pi + 2k\pi}{2}$$

$$k=0; \alpha_1 = \frac{\pi}{2} \longrightarrow \frac{3\pi}{2}$$

$$k=1; \alpha_2 = \frac{\pi + 2\pi}{2} = \frac{3\pi}{2} \longrightarrow \frac{3 \cdot \frac{3\pi}{2}}$$



$$b) \sqrt[3]{-27} = \sqrt[3]{27}i$$

$$r = \sqrt[3]{27} = 3$$

$$\beta = \pi$$

$$\alpha = \frac{\pi + 2k\pi}{3} = \frac{\pi + 2k\pi}{3}$$

$$k=0; \alpha_1 = \frac{\pi}{3} \longrightarrow \frac{3\pi}{3}$$

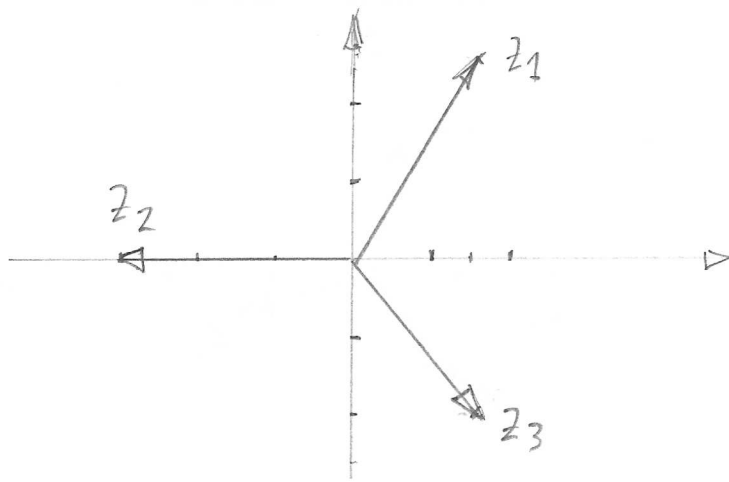
$$k=1; \alpha_2 = \frac{\pi + 2\pi}{3} = \frac{3\pi}{3} = \pi \longrightarrow 3\pi$$

$$k=2; \alpha_3 = \frac{\pi + 4\pi}{3} = \frac{5\pi}{3} \longrightarrow \frac{3 \cdot 5\pi}{3}$$

$$z_1 = 3 \frac{\pi}{3} = 3 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) = 3 \left(\frac{1}{2} + \frac{\sqrt{3}}{2} i \right) = \frac{3}{2} + \frac{3\sqrt{3}}{2} i$$

$$z_2 = 3\pi = 3 (\cos \pi + i \sin \pi) = 3(-1 + 0) = -3$$

$$z_3 = 3 \frac{5\pi}{3} = 3 \left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right) = 3 \left(\frac{1}{2} - \frac{\sqrt{3}}{2} i \right) = \frac{3}{2} - \frac{3\sqrt{3}}{2} i$$



$$c) \sqrt[3]{2-2i}$$

$$z = 2-2i$$

$$|z| = \sqrt{2^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8}$$

$$\beta = \arctg \frac{-2}{2} = \arctg(-1) = -45^\circ = 315^\circ = \frac{7\pi}{4}$$

$$\left. \begin{array}{l} \sqrt{8} \end{array} \right\} \frac{7\pi}{4}$$

$$\sqrt[3]{2-2i} = \sqrt[3]{\sqrt{8} \frac{7\pi}{4}}$$

$$\sqrt[3]{\sqrt{8}} = \sqrt[6]{8} = \sqrt[6]{2^3} = \sqrt{2}$$

$$\beta = \left(\frac{7\pi}{4} \right)$$

$$\alpha = \frac{\beta + 2k\pi}{n} = \frac{\frac{7\pi}{4} + 2k\pi}{3}$$

REVISAR

$$k=0; \alpha_1 = \frac{7\pi}{12}$$

$$\rightarrow \sqrt{2} \frac{7\pi}{12}$$

$$k=1; \alpha_2 = \frac{\frac{7\pi}{4} + 2\pi}{3} = \frac{15\pi}{12} = \frac{5\pi}{4}$$

$$\rightarrow \sqrt{2} \frac{5\pi}{4}$$

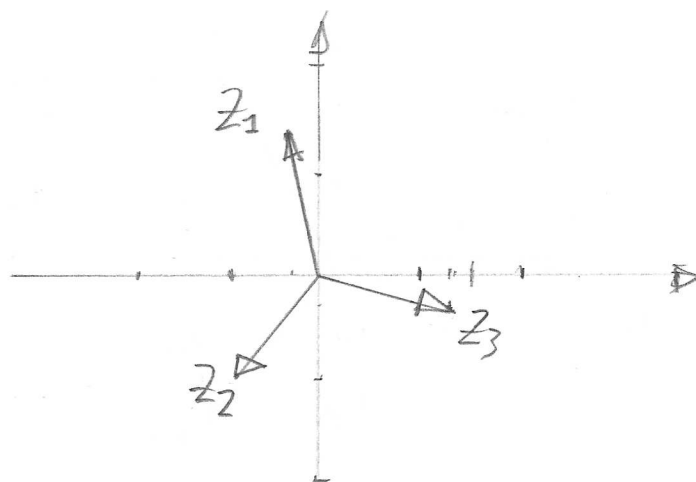
$$k=2; \alpha_3 = \frac{\frac{7\pi}{4} + 4\pi}{3} = \frac{23\pi}{12}$$

$$\rightarrow \sqrt{2} \frac{23\pi}{12}$$

$$z_1 = \sqrt{2} \left(\cos \frac{7\pi}{12} + i \sin \frac{7\pi}{12} \right) = -0,37 + 1,37i$$

$$z_2 = \sqrt{2} \left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} \right) = -1 - i$$

$$z_3 = \sqrt{2} \left(\cos \frac{23\pi}{12} + i \sin \frac{23\pi}{12} \right) = 1,37 - 0,37i$$



$$d) \sqrt[3]{\frac{1-i}{1+i}}$$

Operamos el radicando:

$$\begin{aligned} \frac{1-i}{1+i} &= \frac{(1-i)(1-i)}{(1+i)(1-i)} = \frac{(1-i)^2}{1+1} = \frac{1-2i+i^2}{2} = \\ &= \frac{\cancel{1-2i-1}}{2} = -\frac{2i}{2} = \boxed{-i} \end{aligned}$$

$$\sqrt[3]{\frac{1-i}{1+i}} = \sqrt[3]{-i} = \sqrt[3]{1 \angle 270^\circ} = \sqrt[3]{1 \angle \frac{3\pi}{2}}$$

$$r = \sqrt[3]{1} = \boxed{1}$$

$$\beta = \frac{3\pi}{2} = -\frac{\pi}{2} \quad \text{PROGRAMA}$$

$$\alpha = \frac{\beta + 2k\pi}{3}$$

$$k=0; \alpha_1 = \frac{\frac{3\pi}{2} + 0}{3} = \frac{\frac{3\pi}{2}}{3} = \frac{\pi}{2} \rightarrow 1 + \frac{\pi}{2} \quad \left| \begin{array}{l} 1 - \frac{\pi}{2} \\ 1 + \frac{\pi}{2} \end{array} \right.$$

$$k=1; \alpha_2 = \frac{\frac{3\pi}{2} + 2\pi}{3} = \frac{\frac{7\pi}{2}}{3} = \frac{7\pi}{6} \rightarrow 1 + \frac{7\pi}{6} \quad \left| \begin{array}{l} 1 - \frac{7\pi}{6} \\ 1 + \frac{7\pi}{6} \end{array} \right.$$

$$k=2; \alpha_3 = \frac{\frac{3\pi}{2} + 4\pi}{3} = \frac{\frac{11\pi}{2}}{3} = \frac{11\pi}{6} \rightarrow 1 + \frac{11\pi}{6} \quad \left| \begin{array}{l} 1 - \frac{11\pi}{6} \\ 1 + \frac{11\pi}{6} \end{array} \right.$$

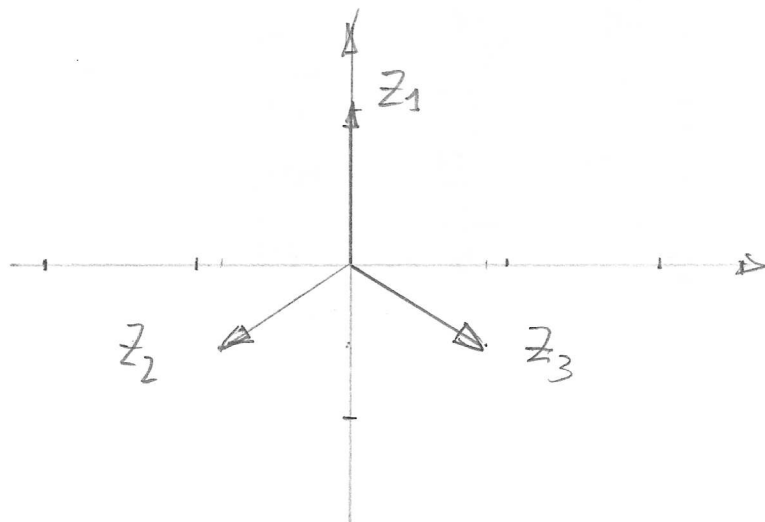
PROGRAMA



$$z_1 = 1 \frac{\pi}{2} = 1 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right) = 1 (0 + i1) = i$$

$$z_2 = 1 \frac{7\pi}{6} = 1 \left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} \right) = 1 \left(-\frac{\sqrt{3}}{2} - i \frac{1}{2} \right) = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$$

$$z_3 = 1 \frac{11\pi}{6} = 1 \left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right) = 1 \left(\frac{\sqrt{3}}{2} - i \frac{1}{2} \right) = \frac{\sqrt{3}}{2} - \frac{1}{2}i$$



e) $\sqrt[5]{-\frac{32}{i}}$ =

Operamos el radicando:

$$\frac{-32}{i} = \frac{-32(-i)}{i(-i)} = \frac{32i}{+1} = 32i \stackrel{\text{POLAR}}{=} 32 \frac{\pi}{2}$$

Por otra parte si lo paramos a polar:

$$\frac{-32}{i} = \frac{32\pi}{1 \frac{\pi}{2}} = (32)\pi - \frac{\pi}{2} = 32\frac{\pi}{2}$$

DA LO MISMO

$$\sqrt[5]{-\frac{32}{i}} = \sqrt[5]{32 \frac{\pi}{2}}$$

$$r = \sqrt[5]{32} = \boxed{2}$$

$$\beta = \frac{\pi}{2}$$

$$\alpha = \frac{\beta + 2k\pi}{n} = \frac{\frac{\pi}{2} + 2k\pi}{5}$$

$$\begin{aligned}
 k=0; \alpha_1 &= \frac{\frac{4\pi}{2}}{5} = \frac{4\pi}{10} \longrightarrow 2 \frac{4\pi}{10} \\
 k=1; \alpha_2 &= \frac{\frac{4\pi}{2} + 2\pi}{5} = \frac{5\pi}{5} = \frac{5\pi}{10} = \frac{4\pi}{2} \longrightarrow 2 \frac{\pi}{2} \\
 k=2; \alpha_3 &= \frac{\frac{4\pi}{2} + 4\pi}{5} = \frac{9\pi/2}{5} = \frac{9\pi}{10} \longrightarrow 2 \frac{9\pi}{10} \\
 k=3; \alpha_4 &= \frac{\frac{4\pi}{2} + 6\pi}{5} = \frac{13\pi/2}{5} = \frac{13\pi}{10} \longrightarrow 2 \frac{13\pi}{10} \\
 k=4; \alpha_5 &= \frac{\frac{4\pi}{2} + 8\pi}{5} = \frac{17\pi/2}{5} = \frac{17\pi}{10} \longrightarrow 2 \frac{17\pi}{10}
 \end{aligned}$$

$$4) \sqrt[3]{8i} = \sqrt[3]{8 \cdot \frac{1}{2}}$$

$$r = \sqrt[3]{8} = \boxed{2}$$

$$\beta = \frac{\pi}{2}$$

$$\alpha = \frac{\beta + 2k\pi}{n} =$$

$$k=0; \alpha_1 = \frac{\frac{\pi}{2} + 0}{3} = \frac{\pi}{6} \rightarrow 2 \frac{\pi}{6}$$

$$k=1; \alpha_2 = \frac{\frac{\pi}{2} + 2\pi}{3} = \frac{\frac{5\pi}{2}}{3} = \frac{5\pi}{6} \rightarrow 2 \frac{5\pi}{6}$$

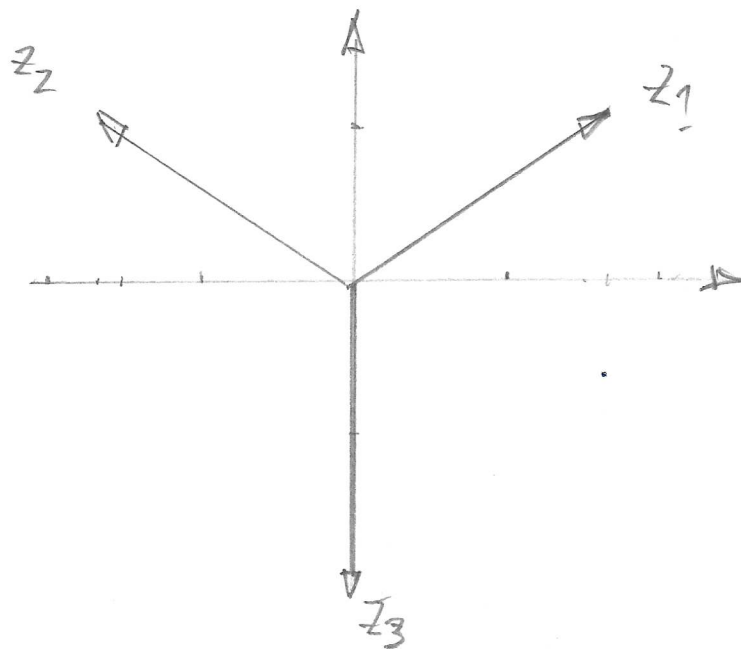
$$k=2; \alpha_3 = \frac{\frac{\pi}{2} + 4\pi}{3} = \frac{\frac{9\pi}{2}}{3} = \frac{9\pi}{6} = \frac{3\pi}{2} \rightarrow 2 \frac{3\pi}{2}$$

Passando a forma binómica:

$$z_1 = 2 \frac{\pi}{6} = 2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) = 2 \left(\frac{\sqrt{3}}{2} + i \frac{1}{2} \right) = \sqrt{3} + i$$

$$z_2 = 2 \frac{5\pi}{6} = 2 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right) = 2 \left(-\frac{\sqrt{3}}{2} + i \frac{1}{2} \right) = -\sqrt{3} + i$$

$$z_3 = 2 \frac{3\pi}{2} = 2 \left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right) = 2(0 - i) = -2i$$



Ej. 40. p 164. ANAYA. BACKILLER 1º

Una de las raíces cúbicas de un w complejo $\neq$ es $1+i$. Halla z y las otras raíces cúbicas.

$$\sqrt[3]{z} = 1+i \quad ; \quad z = (1+i)^3$$

Una raíz

$$w = 1+i$$

$$|w| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\beta = \arctg \frac{1}{1} = \arctg(1) = 45^\circ = \frac{\pi}{4}$$

$$\boxed{\sqrt{2} \frac{\pi}{4}}$$

Las otras raíces cúbicas:

$$\alpha = \frac{\beta + 2k\pi}{3} = \frac{\frac{\pi}{4} + 2k\pi}{3}$$

$$k=0 \quad ; \quad \alpha_1 = \frac{\pi}{4}$$

$$k=1 \quad ; \quad \alpha_2 = \frac{\frac{3\pi}{4} + 2\pi}{3} = \frac{\frac{11\pi}{4}}{3} = \frac{11\pi}{12} \rightarrow \sqrt{2} \frac{11\pi}{12}$$

$$k=2 \quad ; \quad \alpha_3 = \frac{\frac{7\pi}{4} + 4\pi}{3} = \frac{\frac{19\pi}{4}}{3} = \frac{19\pi}{12} \rightarrow \sqrt{2} \frac{19\pi}{12}$$

El complejo es:

$$z = (1+i)^3 = \left(\sqrt{2} \frac{\pi}{4}\right)^3 = \left(\sqrt{2}\right)^3 \frac{3\pi}{4} = \sqrt{8} \frac{3\pi}{4}$$

$$\sqrt{8} \frac{3\pi}{4} = \sqrt{8} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) = \sqrt{8} \left(-\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \right) =$$

$$= \boxed{-2 + 2i}$$

COMPROBACIÓN

$$\sqrt[3]{-2+2i}$$

$$z = -2+2i$$

$$|z| = \sqrt{(-2)^2 + 2^2} = \sqrt{8}$$

$$\beta = \arctan(-1) = 135^\circ = \frac{3\pi}{4}$$

luego:

$$\sqrt[3]{-2+2i} = \sqrt[3]{\sqrt{8} \frac{3\pi}{4}}$$

$$r = \sqrt[3]{\sqrt{8}} = \sqrt[6]{8} = \sqrt[6]{2^3} = \sqrt{2}$$

$$\alpha = \frac{\beta + 2k\pi}{n} = \frac{3\pi/4 + 2k\pi}{3}$$

ya se ve que es correcto.

2, p151

Obten polinomios cuyas raíces sean:

a) $2 + \sqrt{3}i$ y $2 - \sqrt{3}i$

b) $-3i$ y $3i$

c) $1 + 2i$ y $3 - 4i$

a) $(x - \underbrace{2 + \sqrt{3}i})(x - \underbrace{2 - \sqrt{3}i}) = 0$

$$[x - (2 + \sqrt{3}i)][x - (2 - \sqrt{3}i)] = 0$$

$$= x^2 - x(2 - \sqrt{3}i) - x(2 + \sqrt{3}i) + 7 = 0$$

$$x^2 - 2x + x\sqrt{3}i - 2x - x\sqrt{3}i + 7 = 0$$

$$\boxed{x^2 - 4x + 7 = 0}$$

b) $[x - (-3i)][x - (3i)] = 0$

$$(x + 3i)(x - 3i) = 0$$

$$\boxed{x^2 + 9 = 0}$$

c) $[x - (1 + 2i)][x - (3 - 4i)] = 0$

$$x^2 - x(3 - 4i) - x(1 + 2i) + (1 + 2i)(3 - 4i) = 0$$

$$x^2 - 3x + 4xi - x - 2xi + (3 - 4i + 6i + 8) = 0$$

$$x^2 - 4x + 2xi + 2i + 11 = 0$$

$$x^2 - 4x + 11 + 2i(x + 1) = 0$$

SOLO CUANDO LAS DOS RAÍCES SON CONJUGADAS, EL POLINOMIO TIENE RAÍCES REALES.

3, p 151

¿Cuánto debe valer x , real, para que $(25 - xi)^2$ sea imaginario puro?

$$(25 - xi)^2 = 625 - 50xi - x^2$$

Para que sea imaginario puro:

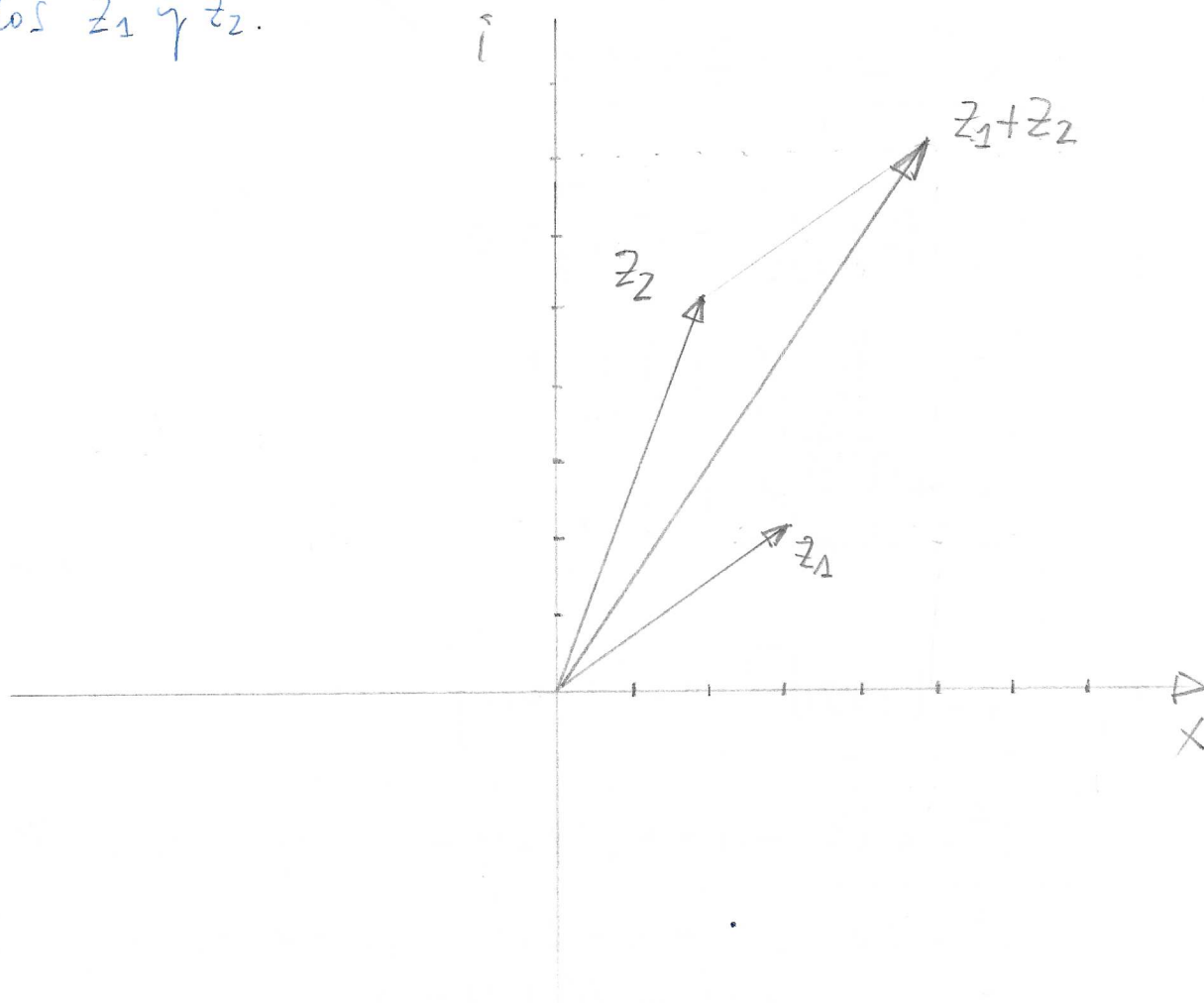
$$625 - x^2 = 0 ; \quad x^2 = 625 ; \quad x = \pm \sqrt{625} = \pm 25$$

Hay dos soluciones:

$$x_1 = 25, \quad x_2 = -25$$

4, p 151

Representa gráficamente: $z_1 = 3 + 2i$, $z_2 = 2 + 5i$ y $z_1 + z_2$.
Comprueba que $z_1 + z_2$ es una diagonal del paralelogramo de lados z_1 y z_2 .



33, #84

Calcular las soluciones de la ecuación:

$$z^4 + (1+i)z^2 + 5i = 0$$

Haciendo el cambio $w = z^2$

$$w^2 + (1+i)w + 5i = 0$$

$$w = \frac{-(1+i) \pm \sqrt{(1+i)^2 - 20i}}{2} = \frac{-(1+i) \pm \sqrt{-18i}}{2}$$

$$= \frac{-(1+i) \pm 3\sqrt{2} \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i \right)}{2} = \frac{-(1+i) \pm 3(1-i)}{2} =$$

$$\rightarrow \frac{-(1+i) + 3(1-i)}{2} = \frac{-1-i+3-3i}{2} = \frac{2-4i}{2} = \underline{\underline{1-2i}}$$

$$\searrow \frac{-(1+i) - 3(1-i)}{2} = \frac{-1-i-3+3i}{2} = \frac{-4+2i}{2} = \underline{\underline{-2+i}}$$

$$\text{Arg}(1-2i) = \arctg(-2) = -\arctg(2)$$

$$\text{Arg}(-2+i) = \arctg\left(-\frac{1}{2}\right) = -\arctg\left(\frac{1}{2}\right)$$

Las raíces son:

$$\pm \sqrt[4]{1-2i} = \pm \sqrt[4]{5} \left(\cos\left(\frac{\arctg 2}{2}\right) - i \sin\left(\frac{\arctg 2}{2}\right) \right)$$

$$\pm \sqrt[4]{-2+i} = \pm \sqrt[4]{5} \left(\cos\left(\frac{\arctg(1/2)}{2}\right) + i \sin\left(\frac{\arctg(1/2)}{2}\right) \right)$$

$$(1+i)^2 = 1 + 2i - 1 = 2i$$

$$\sqrt{-18i} = \sqrt{18} \left(\cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right) \right) = 3\sqrt{2} \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i \right) =$$

$$= 3\sqrt{2} \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i \right)$$

$-45^\circ = 315^\circ$

34, p 24

Calcular las soluciones de las ecuaciones:

a) $z^4 + 2z^3 + 7z^2 - 18z + 26 = 0$

b) $z^4 + (5+4i)z^2 + 10i = 0$

a) El $u = (1+i)$ es raíz $P(x)$

$P(x)$ se anula para $x = \alpha$

$$P(\alpha) = 0 \Rightarrow P(x) = (x - \alpha) Q(x)$$

Si $(1+i)$ es raíz, también lo es $(1-i)$

$$(z - (1+i))(z - (1-i)) = z^2 - 2z + 2$$

Haciendo la división:

$$\begin{array}{r|l} z^4 + 2z^3 + 7z^2 - 18z + 26 & z^2 - 2z + 2 \\ \hline & z^2 + 4z + 13 \end{array}$$

luego:

$$z^4 + 2z^3 + 7z^2 - 18z + 26 = (z^2 - 2z + 2)(z^2 + 4z + 13) = 0$$

$$z^2 - 2z + 2 = 0$$

$$z_1 = 1-i$$

$$z_2 = 1+i$$

$$z^2 + 4z + 13 = 0$$

$$z_3 = -2+3i$$

$$z_4 = -2-3i$$

Resuelto con
MATHEMATICA

4.- Resuelve las ecuaciones:

a) $x^4 + 1 = 0$

b) $x^6 + 64 = 0$

a) $x^4 + 1 = 0$; $x^4 = -1$; $x = \sqrt[4]{-1}$

$$\sqrt[4]{-1} = \sqrt[4]{1 \angle 180^\circ} = \sqrt[4]{1 \angle \pi}$$

$$r = \sqrt[4]{1} = \boxed{1}$$

$$\beta = \pi$$

$$\alpha = \frac{\beta + 2k\pi}{n} = \frac{\pi + 2k\pi}{4}$$

$$k=0 ; \alpha_1 = \frac{\pi}{4} \rightarrow 1 \angle \frac{\pi}{4}$$

$$k=1 ; \alpha_2 = \frac{\pi + 2\pi}{4} = \frac{3\pi}{4} \rightarrow 1 \angle \frac{3\pi}{4}$$

$$k=2 ; \alpha_3 = \frac{\pi + 4\pi}{4} = \frac{5\pi}{4} \rightarrow 1 \angle \frac{5\pi}{4}$$

$$k=3 ; \alpha_4 = \frac{\pi + 6\pi}{4} = \frac{7\pi}{4} \rightarrow 1 \angle \frac{7\pi}{4}$$

b) $x^6 + 64 = 0$; $x^6 = -64$; $x = \sqrt[6]{-64}$

$$\sqrt[6]{-64} = \sqrt[6]{64 \angle \pi}$$

$$r = \sqrt[6]{64} = \boxed{2}$$

$$\beta = \pi$$

$$\alpha = \frac{\beta + 2k\pi}{n} = \frac{\pi + 2k\pi}{6}$$

$$k=0 ; \alpha_1 = \frac{\pi}{6} \rightarrow 2 \angle \frac{\pi}{6}$$

$$k=1 ; \alpha_2 = \frac{\pi + 2\pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2} \rightarrow 2 \angle \frac{\pi}{2}$$

$$k=2 ; \alpha_3 = \frac{\pi + 4\pi}{6} = \frac{5\pi}{6} \rightarrow 2 \angle \frac{5\pi}{6}$$

$$k=3; \alpha_4 = \frac{n+6n}{6} = \frac{7n}{6} \rightarrow 2 \frac{7n}{6}$$

$$k=4; \alpha_5 = \frac{n+8n}{6} = \frac{9n}{6} = \frac{3n}{2} \rightarrow 2 \frac{3n}{2}$$

$$k=5; \alpha_6 = \frac{n+10n}{6} = \frac{11n}{6} \rightarrow 2 \frac{11n}{6}$$

5.- Comprueba que si z y w son las raíces sextas de 1 entonces también lo son los resultados de las siguientes operaciones:

$$z \cdot w, \quad z/w, \quad z^2, \quad z^3$$

$$z^6 = 1 \quad w^6 = 1$$

$$(z \cdot w)^6 = z^6 \cdot w^6 = 1 \cdot 1 = 1$$

$$\Rightarrow z \cdot w = \sqrt[6]{1} = 1$$

$$\left(\frac{z}{w}\right)^6 = \frac{z^6}{w^6} = \frac{1}{1} = 1$$

$$\Rightarrow \frac{z}{w} = \sqrt[6]{1} = 1$$

$$(z^2)^6 = z^{12} = (z^4)^3 = 1^3 = 1$$

$$\Rightarrow z^2 = \sqrt[6]{1} = 1$$

$$(z^3)^6 = z^{18} = z^{16} z^2 = (z^4)^4 \cdot z^2 = 1^4 \cdot 1^2 = 1$$

$$\Rightarrow z^3 = \sqrt[6]{1} = 1$$

2, p 149

Resuelva y representa las soluciones:

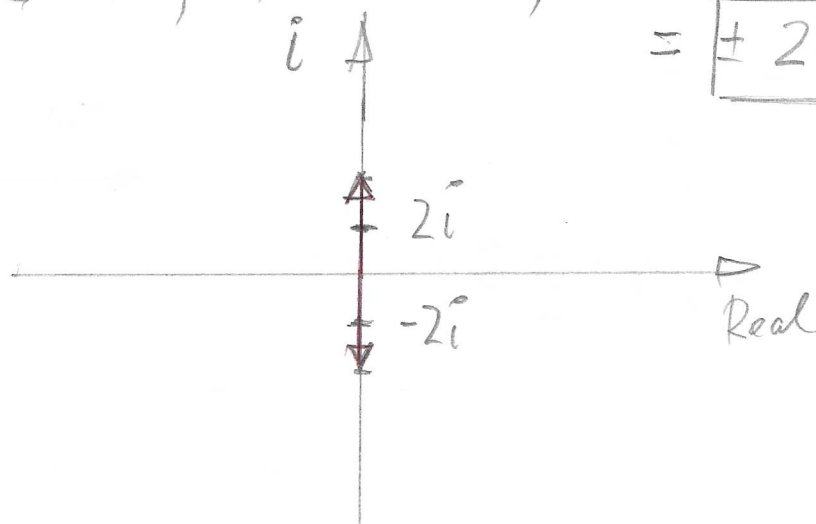
a) $x^2 + 4 = 0$

b) $x^2 + 6x + 10 = 0$

c) $3x^2 + 27 = 0$

d) $3x^2 - 27 = 0$

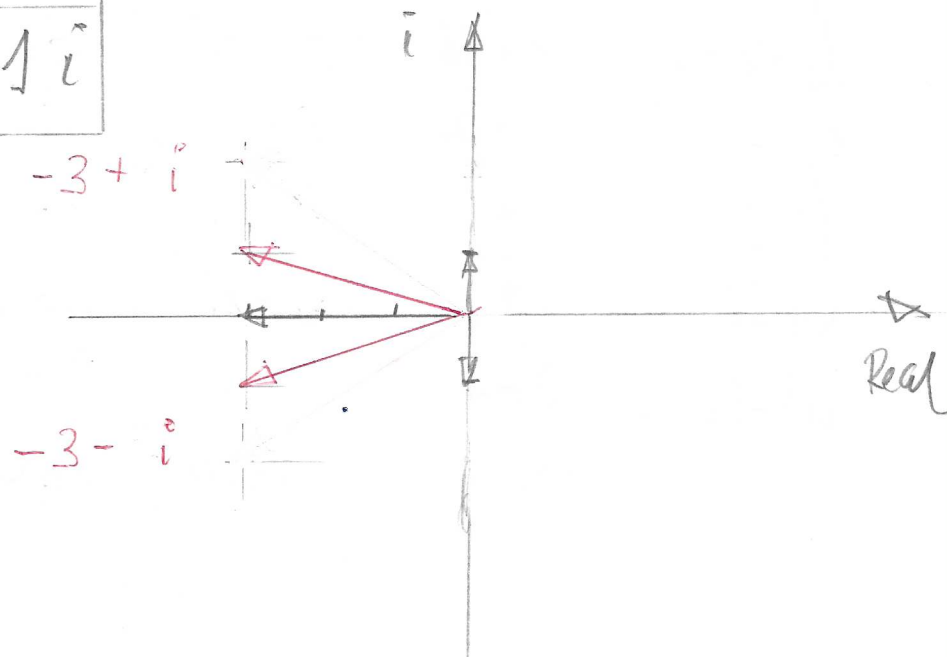
a) $x^2 + 4 = 0$; $x^2 = -4$; $x = \pm \sqrt{-4} = \pm \sqrt{4} \sqrt{-1} = \pm 2i$



b) $x^2 + 6x + 10 = 0$

$$x = \frac{-6 \pm \sqrt{36 - 40}}{2} = \frac{-6 \pm \sqrt{-4}}{2} = \frac{-6 \pm \sqrt{4} \sqrt{-1}}{2} =$$

$$= \boxed{-3 \pm 1i}$$

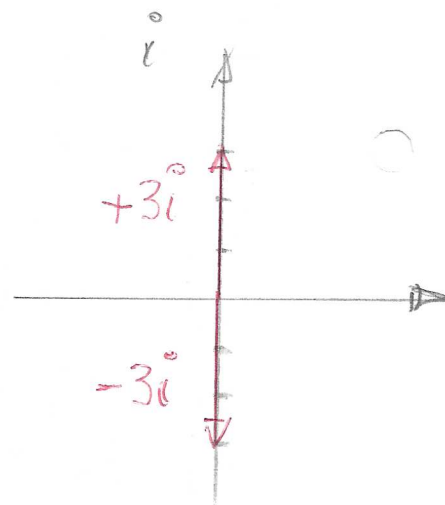


$$c) 3x^2 + 27 = 0$$

$$3x^2 = -27$$

$$x^2 = -\frac{27}{3} = -9$$

$$x = \pm \sqrt{-9} = \pm \sqrt{9} \sqrt{-1} = \boxed{\pm 3i}$$

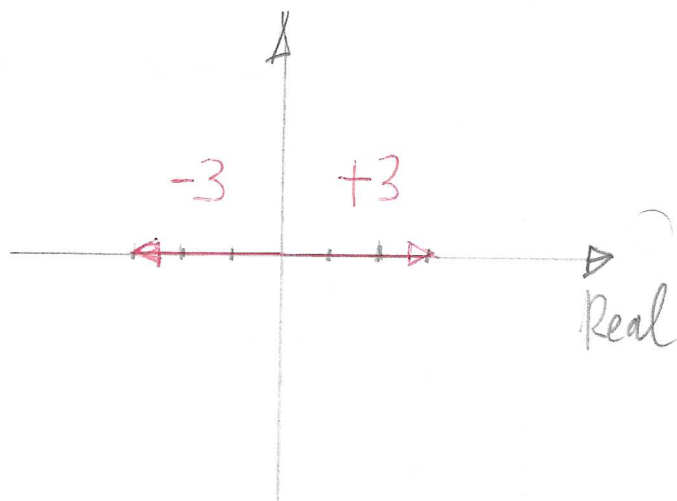


$$d) 3x^2 - 27 = 0$$

$$3x^2 = 27$$

$$x^2 = \frac{27}{3} = 9$$

$$x = \pm \sqrt{9} = \boxed{\pm 3}$$



3, +149

Representar gráficamente el opuesto y el conjugado de:

a) $3-5i$	$3-5i$	$-3+5i$	$3+5i$
b) $5+2i$	$5+2i$	$-5-2i$	$5-2i$
c) $-1-2i$	$-1-2i$	$1+2i$	$-1+2i$
d) $-2+3i$	$-2+3i$	$2-3i$	$-2-3i$
e) 5	$5+0i$	$-5-0i$	$5-0i$
f) 0	$0+0i$	$0-0i$	$0-0i$
g) $2i$	$0+2i$	$0-2i$	$0-2i$
h) $-5i$	$0-5i$	$+5i$	$0+5i$

COMPLETO

OPUESTO

CONJUGADO

L1, p164. ANAYA. BACHILLER 1º

Resuelve las ecuaciones y expresa el resultado en forma binómica.

a) $x^2 + 4 = 0$

b) $x^2 + x + 4 = 0$

c) $x^2 + 3x + 7 = 0$

d) $x^2 - x + 1 = 0$

a) $x^2 + 4 = 0$; $x^2 = -4$; $x = \pm \sqrt{-4} = \pm \sqrt{4} \cdot \sqrt{-1} = \boxed{\pm 2i}$

Haciendo por raíces:

$$\pm \sqrt{-4} = \pm \sqrt{-4 + 0i} = \sqrt{4} \pi$$

$$r = \sqrt{4} = 2$$

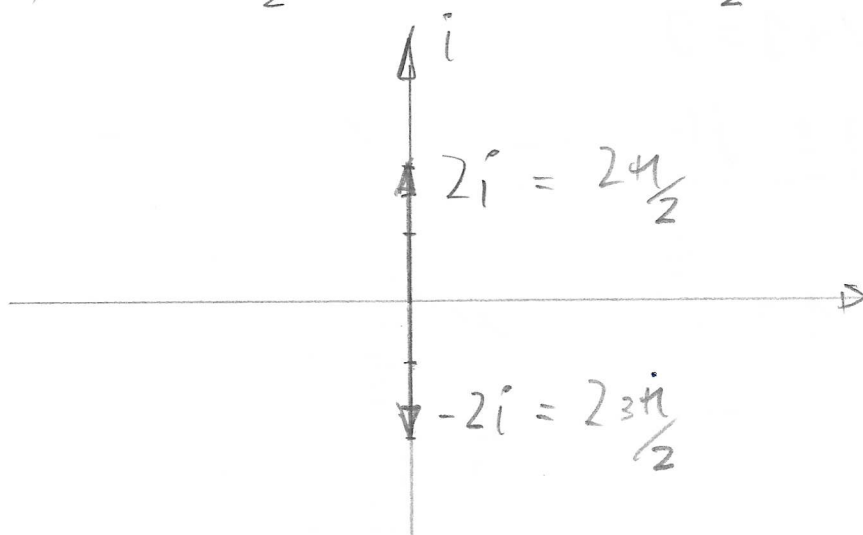
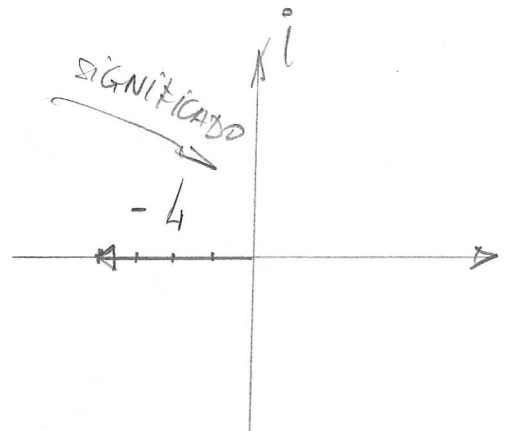
$$\beta = \operatorname{arctg} \frac{0}{-4} = \operatorname{arctg}(0) = \pi$$

Las raíces son:

$$\alpha = \frac{\beta + 2k\pi}{n} = \frac{\pi + 2k\pi}{2}$$

$$k=0 ; \alpha_1 = \frac{\pi}{2} \rightarrow 2\pi/2$$

$$k=1 ; \alpha_2 = \frac{\pi + 2\pi}{2} = \frac{3\pi}{2} \rightarrow 2 \cdot 3\pi/2$$



$$b) x^2 + x + 4 = 0$$

$$\begin{aligned}
 x &= \frac{-1 \pm \sqrt{1^2 - 4 \times 4}}{2} = \frac{-1 \pm \sqrt{1 - 16}}{2} = \frac{-1 \pm \sqrt{-15}}{2} = \\
 &= \frac{-1 \pm \sqrt{15}(-1)}{2} = \frac{-1 \pm \sqrt{15} \sqrt{-1}}{2} = \frac{-1 \pm \sqrt{15} i}{2} = \\
 &= \begin{cases} -\frac{1}{2} + \frac{\sqrt{15}}{2} i \\ -\frac{1}{2} - \frac{\sqrt{15}}{2} i \end{cases}
 \end{aligned}$$

$$c) x^2 + 3x + 7 = 0$$

$$\begin{aligned}
 x &= \frac{-3 \pm \sqrt{3^2 - 4 \times 7}}{2} = \frac{-3 \pm \sqrt{9 - 28}}{2} = \frac{-3 \pm \sqrt{-19}}{2} = \\
 &= \frac{-3 \pm \sqrt{19} i}{2} \rightarrow \begin{cases} -\frac{3}{2} + \frac{\sqrt{19}}{2} i \\ -\frac{3}{2} - \frac{\sqrt{19}}{2} i \end{cases}
 \end{aligned}$$

$$d) x^2 - x + 1 = 0$$

$$\begin{aligned}
 x &= \frac{1 \pm \sqrt{(-1)^2 - 4}}{2} = \frac{1 \pm \sqrt{1 - 4}}{2} = \frac{1 \pm \sqrt{-3}}{2} = \\
 &= \frac{1 \pm \sqrt{3} i}{2} \rightarrow \begin{cases} \frac{1}{2} + \frac{\sqrt{3}}{2} i \\ \frac{1}{2} - \frac{\sqrt{3}}{2} i \end{cases}
 \end{aligned}$$

Ej. 42, p 164. ANAYA. BACHILLER 1º

Resuelve las ecuaciones:

a) $x^5 + 32 = 0$

b) $i x^3 - 27 = 0$

a) $x^5 + 32 = 0$; $x^5 = -32$; $x = \sqrt[5]{-32}$

$$\sqrt[5]{-32} = \sqrt[5]{32 \cdot 180^\circ} = \sqrt[5]{32} \pi$$

$$r = \sqrt[5]{32} = 2$$

$$\beta = \pi$$

Los argumentos son:

$$\alpha = \frac{\beta + 2k\pi}{n} = \frac{\pi + 2k\pi}{5}$$

$$k=0 ; \alpha_1 = \frac{\pi}{5} \rightarrow 2 \frac{\pi}{5}$$

$$k=1 ; \alpha_2 = \frac{\pi + 2\pi}{5} = \frac{3\pi}{5} \rightarrow 2 \frac{3\pi}{5}$$

$$k=2 ; \alpha_3 = \frac{\pi + 4\pi}{5} = \frac{5\pi}{5} = \pi \rightarrow 2\pi$$

$$k=3 ; \alpha_4 = \frac{\pi + 6\pi}{5} = \frac{7\pi}{5} \rightarrow 2 \frac{7\pi}{5}$$

$$k=4 ; \alpha_5 = \frac{\pi + 8\pi}{5} = \frac{9\pi}{5} \rightarrow 2 \frac{9\pi}{5}$$

b) $i x^3 - 27 = 0$; $x^3 = \frac{27}{i} = \frac{27 \cdot i}{i \cdot i} = -27i$

$$x = \sqrt[3]{-27i} = \sqrt[3]{27 \frac{3\pi}{2}} = \sqrt[3]{27} \left(-\frac{\pi}{2}\right)$$

INTERPRETACIÓN

$$r = \sqrt[3]{27} = 3$$

$$\beta = \frac{3\pi}{2}$$

Las raíces son:

$$\alpha = \frac{\beta + 2k\pi}{n} = \frac{\frac{3\pi}{2} + 2k\pi}{3}$$

$$k=0; \alpha_1 = \frac{\frac{3\pi}{2}}{3} = \frac{\pi}{2} \rightarrow 3 \frac{\pi}{2}$$

$$k=1; \alpha_2 = \frac{\frac{3\pi}{2} + 2\pi}{3} = \frac{\frac{7\pi}{2}}{3} = \frac{7\pi}{6} \rightarrow 3 \frac{7\pi}{6}$$

$$k=2; \alpha_3 = \frac{\frac{3\pi}{2} + 4\pi}{3} = \frac{\frac{11\pi}{2}}{3} = \frac{11\pi}{6} \rightarrow 3 \frac{11\pi}{6}$$

Podía haber cogido $\beta = -\frac{\pi}{2}$, entonces las raíces serían:

$$\alpha = \frac{-\frac{\pi}{2} + 2k\pi}{n} = \frac{-\frac{\pi}{2} + 2k\pi}{3}$$

$$k=0; \alpha_1 = -\frac{\pi}{6} \rightarrow 3(-\frac{\pi}{6})$$

$$k=1; \alpha_2 = \frac{-\frac{\pi}{2} + 2\pi}{3} = \frac{\frac{3\pi}{2}}{3} = \frac{3\pi}{6} = \frac{\pi}{2} \rightarrow 3 \frac{\pi}{2}$$

$$k=2; \alpha_3 = \frac{-\frac{\pi}{2} + 4\pi}{3} = \frac{\frac{7\pi}{2}}{3} = \frac{7\pi}{6} \rightarrow 3 \frac{7\pi}{6}$$

Ex. 43. ANAYA. p 164. BACHILLER 1º

Resuelve las ecuaciones:

a) $z^2 + 4 = 0$

b) $z^2 - 2z + 5 = 0$

c) $2z^2 + 10 = 0$

a) $z^2 + 4 = 0$; $z^2 = -4$; $z = \pm \sqrt{-4} = \pm \sqrt{4}i = \pm 2i$

$$\boxed{\begin{array}{l} z_1 = +2i \\ z_2 = -2i \end{array}}$$

b) $z^2 - 2z + 5 = 0$

$$z = \frac{2 \pm \sqrt{(-2)^2 - 20}}{2} = \frac{2 \pm \sqrt{4 - 20}}{2} = \frac{2 \pm \sqrt{-16}}{2} =$$

$$= \frac{2 \pm 4i}{2} \rightarrow \boxed{\begin{array}{l} z_1 = 1 + 2i \\ z_2 = 1 - 2i \end{array}}$$

c) $2z^2 + 10 = 0$; $2z^2 = -10$; $z^2 = -\frac{10}{2} = -5$

$$z = \pm \sqrt{-5} = \pm \sqrt{5}i$$

$$\boxed{\begin{array}{l} z_1 = \sqrt{5}i \\ z_2 = -\sqrt{5}i \end{array}}$$

Ej. 44, p. 164. ANAYA. BACHILLER 1º

Obtén las soluciones de las siguientes ecuaciones:

a) $z^4 - 1 = 0$

b) $z^4 + 16 = 0$

c) $z^4 - 8z = 0$

a) $z^4 - 1 = 0$; $z^4 = 1$; $z = \sqrt[4]{1}$

$$z = \sqrt[4]{1} = \sqrt[4]{1 \angle 0^\circ}$$

$$r = \sqrt[4]{1} = 1$$

$$\beta = 0^\circ$$

Las raíces son:

$$\alpha = \frac{\beta + 2k\pi}{n} = \frac{0^\circ + 2k\pi}{4}$$

$$k=0 ; \alpha_1 = \frac{0}{4} = 0^\circ \rightarrow 1 \angle 0^\circ = 1$$

$$k=1 ; \alpha_2 = \frac{2\pi}{4} = \frac{\pi}{2} \rightarrow 1 \angle \frac{\pi}{2} = i$$

$$k=2 ; \alpha_3 = \frac{4\pi}{4} = \pi \rightarrow 1 \angle \pi = -1$$

$$k=3 ; \alpha_4 = \frac{6\pi}{4} = \frac{3\pi}{2} \rightarrow 1 \angle \frac{3\pi}{2} = -i$$

b) $z^4 + 16 = 0$; $z^4 = -16$; $z = \sqrt[4]{-16}$

$$\sqrt[4]{-16} = \sqrt[4]{16 \angle \pi}$$

$$r = \sqrt[4]{16} = 2$$

$$\beta = \pi$$

Las raíces son:

$$\alpha = \frac{\beta + 2k\pi}{n} = \frac{\pi + 2k\pi}{4}$$

$$k=0; \alpha_1 = \frac{\pi}{4} \rightarrow 2 \frac{\pi}{4} = \sqrt{2} + \sqrt{2}i$$

$$k=1; \alpha_2 = \frac{\pi + 2\pi}{4} = \frac{3\pi}{4} \rightarrow 2 \frac{3\pi}{4} = -\sqrt{2} + \sqrt{2}i$$

$$k=2; \alpha_3 = \frac{\pi + 4\pi}{4} = \frac{5\pi}{4} \rightarrow 2 \frac{5\pi}{4} = -\sqrt{2} - \sqrt{2}i$$

$$k=3; \alpha_4 = \frac{\pi + 6\pi}{4} = \frac{7\pi}{4} \rightarrow 2 \frac{7\pi}{4} = \sqrt{2} - \sqrt{2}i$$

HECHO CON PROGRAMA

$$c) z^4 - 8z = 0; z(z^3 - 8) = 0 \rightarrow \boxed{z=0} \rightarrow \boxed{0}$$

$$\rightarrow z^3 - 8 = 0$$

$$z^3 - 8 = 0; z^3 = 8; z = \sqrt[3]{8} = \boxed{2}$$

$$z = 2 = 2 \cdot 0^\circ$$

$$l = 2$$

$$\beta = 0^\circ$$

Las raíces son:

$$\alpha = \frac{\beta + 2k\pi}{n} = \frac{0 + 2k\pi}{3}$$

$$k=0; \alpha_1 = \frac{0}{3} = 0^\circ \rightarrow 2 \cdot 0^\circ = 2$$

$$k=1; \alpha_2 = \frac{2\pi}{3} \rightarrow 2 \frac{2\pi}{3} = -1 + \sqrt{3}i$$

$$k=2; \alpha_3 = \frac{4\pi}{3} \rightarrow 2 \frac{4\pi}{3} = -1 - \sqrt{3}i$$

Ej. 45, p 164. ANAYA. BACHILLER 1º

Resuelva estas ecuaciones y exprese las soluciones en f. binómica.

a) $z^3 + 8i = 0$

b) $iz^4 + 4 = 0$

a) $z^3 + 8i = 0$, $z^3 = -8i$, $z = \sqrt[3]{-8i}$

$z = \sqrt[3]{-8i} = \sqrt[3]{8 \cdot 270^\circ} = \sqrt[3]{8} \frac{3\pi}{2}$

$r = \sqrt[3]{8} = \boxed{2}$

$\beta = \frac{3\pi}{2}$

Las raíces son:

$\alpha = \frac{\beta + 2k\pi}{n} = \frac{\frac{3\pi}{2} + 2k\pi}{3}$

$k=0$; $\alpha_1 = \frac{3\pi}{6} = \frac{\pi}{2} \rightarrow 2 \frac{\pi}{2}$

$k=1$; $\alpha_2 = \frac{\frac{3\pi}{2} + 2\pi}{3} = \frac{7\pi}{6} \rightarrow 2 \frac{7\pi}{6}$

$k=2$; $\alpha_3 = \frac{\frac{3\pi}{2} + 4\pi}{3} = \frac{11\pi}{6} \rightarrow 2 \frac{11\pi}{6}$

Binómica

$2 \frac{\pi}{2} = 2 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right) = 2 (0 + i \cdot 1) = 2i$

$2 \frac{7\pi}{6} = 2 \left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} \right) = 2 \left(-\frac{\sqrt{3}}{2} - i \frac{1}{2} \right) = -\sqrt{3} - i$

$2 \frac{11\pi}{6} = 2 \left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right) = 2 \left(\frac{\sqrt{3}}{2} - i \frac{1}{2} \right) = \sqrt{3} - i$

$$b) i z^4 + 4 = 0 ; i z^4 = -4 ; z^4 = -\frac{4}{i} = 4i ; z = \sqrt[4]{4i}$$

$$z = \sqrt[4]{4i} = \sqrt[4]{4 \cdot 90^\circ} = \sqrt[4]{4} \cdot \frac{\pi}{2}$$

$$r = \sqrt[4]{4} = \boxed{\sqrt{2}}$$

$$\beta = \frac{\pi}{2}$$

Las raíces son:

$$\alpha = \frac{\beta + 2k\pi}{n} = \frac{\frac{\pi}{2} + 2k\pi}{4}$$

POLAR

$$k=0 ; \alpha_1 = \frac{\pi}{8} \rightarrow \sqrt{2} \frac{\pi}{8}$$

$$k=1 ; \alpha_2 = \frac{\frac{\pi}{2} + 2\pi}{4} = \frac{5\pi}{8} \rightarrow \sqrt{2} \frac{5\pi}{8}$$

$$k=2 ; \alpha_3 = \frac{\frac{\pi}{2} + 4\pi}{4} = \frac{9\pi}{8} \rightarrow \sqrt{2} \frac{9\pi}{8}$$

$$k=3 ; \alpha_4 = \frac{\frac{\pi}{2} + 6\pi}{4} = \frac{13\pi}{8} \rightarrow \sqrt{2} \frac{13\pi}{8}$$

En forma binómica

$$\sqrt{2} \frac{3\pi}{8} = \sqrt{2} \left(\cos \frac{\pi}{8} + i \sin \frac{\pi}{8} \right) = 1,3 + 0,5i$$

$$\sqrt{2} \frac{7\pi}{8} = \sqrt{2} \left(\cos \frac{5\pi}{8} + i \sin \frac{5\pi}{8} \right) = -0,5 + 1,3i$$

$$\sqrt{2} \frac{11\pi}{8} = \sqrt{2} \left(\cos \frac{9\pi}{8} + i \sin \frac{9\pi}{8} \right) = -1,3 - 0,5i$$

$$\sqrt{2} \frac{15\pi}{8} = \sqrt{2} \left(\cos \frac{13\pi}{8} + i \sin \frac{13\pi}{8} \right) = 0,5 - 1,3i$$

OPUESTO

OPUESTO

Ej. 46, p164. ANAYA. BACHILLER 1º

Escribe una ecuación de segundo grado que tenga por soluciones:

$$1+i \text{ y } 2-3i$$

El polinomio de segundo grado será:

$$(x-a)(x-b)=0$$

$$\left[z - (1+i) \right] \left[z - (2-3i) \right] = 0$$

$$z^2 - z(2-3i) - z(1+i) + (1+i)(2-3i) = 0$$

$$z^2 - 2z + 3zi - z - zi + 2 - 3i + 2i + 3 = 0$$

$$\boxed{z^2 - 2z + 2zi - i + 3 = 0}$$

$$\boxed{z^2 + 2z(-1+i) + 3-i = 0}$$

Las dos formas conducen:

$$z_1 = (1-i) - \sqrt{-3-i}$$

$$z_2 = (1-i) + \sqrt{-3-i}$$

Resuelto con
MATHEMATICA

Ej. 47, p 104. ANAYA. BACHILLER 1º

— Escribe la ecuación de segundo grado cuyas soluciones sean: $(2-3i)$ y $(2+3i)$

El polinomio de segundo grado será:

$$(x-a)(x-b)=0$$

$$[z-(2-3i)][z-(2+3i)]=0$$

$$z^2 - z(2+3i) - z(2-3i) + (2-3i)(2+3i) = 0$$

$$z^2 - 2z - 3zi - 2z + 3zi + 4 + 9 = 0$$

$$\boxed{z^2 - 4z + 13 = 0}$$

Comprobación con MATHEMATICA

FM1. Práctica 1

7.- Resuelve, en el campo complejo, las ecuaciones:

a) $z^4 + 81 = 0$

b) $z^5 + z^3 + z = 0$

c) $z^2 + (2i - 3)z + 5 - i = 0$

a) $z^4 + 81 = 0$; $z^4 = -81$; $z = \sqrt[4]{-81} = \sqrt[4]{81} \pi$
INTERPRETACIÓN EN POLAR

$w = 81 \pi$

$|w| = \sqrt[4]{81} = 3$ }
 $\beta = \pi$

Las raíces son:

$\alpha = \frac{\beta + 2k\pi}{4} = \frac{\pi + 2k\pi}{4}$

$k=0$; $\alpha_1 = \frac{\pi}{4} \rightarrow 3 \frac{\pi}{4}$

$k=1$; $\alpha_2 = \frac{\pi + 2\pi}{4} = \frac{3\pi}{4} \rightarrow 3 \frac{3\pi}{4}$

$k=2$; $\alpha_3 = \frac{\pi + 4\pi}{4} = \frac{5\pi}{4} \rightarrow 3 \frac{5\pi}{4}$

$k=3$; $\alpha_4 = \frac{\pi + 6\pi}{4} = \frac{7\pi}{4} \rightarrow 3 \frac{7\pi}{4}$

$z_1 = 3 \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) = \frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2} i$

$z_2 = 3 \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) = -\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2} i$

$z_3 = 3 \left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} \right) = -\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2} i$

$z_4 = 3 \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right) = \frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2} i$

$$b) \quad z^5 + z^3 + z = 0 \quad ; \quad z(z^4 + z^2 + 1) = 0$$

primera solución: $\boxed{z_1 = 0}$

las demás salen:

$$z^4 + z^2 + 1 = 0$$

$$z^2 = \frac{-1 \pm \sqrt{1^2 - 4}}{2} = \frac{-1 \pm \sqrt{-3}}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

Si $z^2 = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$ entonces:

$$z = \pm \left[\sqrt{\frac{1}{2}\left(1 - \frac{1}{2}\right)} + i \sqrt{\frac{1}{2}\left(1 + \frac{1}{2}\right)} \right] = \pm \left(\frac{1}{2} + \frac{\sqrt{3}}{2} i \right)$$

Si $z^2 = -\frac{1}{2} - \frac{\sqrt{3}}{2} i$ entonces:

$$z = \pm \left[\sqrt{\frac{1}{2}\left(1 - \frac{1}{2}\right)} - i \sqrt{\frac{1}{2}\left(1 + \frac{1}{2}\right)} \right] = \pm \left(\frac{1}{2} - \frac{\sqrt{3}}{2} i \right)$$

$$c) \quad z^2 + (2i - 3)z + (5 - i) = 0$$

$$z = \frac{-2i + 3 \pm \sqrt{(2i - 3)^2 - 4(5 - i)}}{2} = \frac{-2i + 3 \pm \sqrt{-15 - 8i}}{2} =$$

Calculamos $\sqrt{-15 - 8i}$

$$\sqrt{-15 - 8i} = \pm \left[\sqrt{\frac{1}{2}(17 - 15)} - i \sqrt{\frac{1}{2}(17 + 15)} \right] = \pm (1 - 4i)$$

$$= \frac{-2i + 3 \pm (1 - 4i)}{2} = \begin{cases} \frac{-2i + 3 + 1 - 4i}{2} = \frac{4 - 6i}{2} = \boxed{2 - 3i} \\ \frac{-2i + 3 - 1 + 4i}{2} = \frac{2 + 2i}{2} = \boxed{1 + i} \end{cases}$$

T6. N° COMPLEJOS. BACHILLER 1º. ANAYA

p147

Resuelve las siguientes ecuaciones:

d) $x^2 + 1 = 0$

e) $x^2 - 2x + 5 = 0$

f) $5x^2 + 10 = 0$

d) $x^2 + 1 = 0$; $x^2 = -1$; $x = \pm \sqrt{-1} = \boxed{\pm i}$

e) $x^2 - 2x + 5 = 0$

$$x = \frac{2 \pm \sqrt{4 - 20}}{2} = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm \sqrt{16} \sqrt{-1}}{2} =$$
$$= \boxed{1 \pm 2i}$$

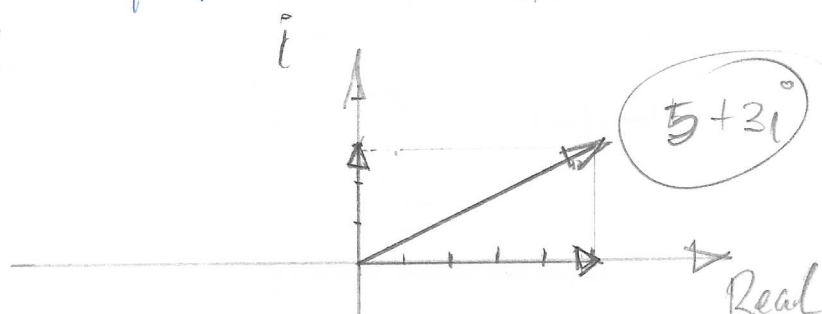
f) $5x^2 + 10 = 0$; $5x^2 = -10$; $x^2 = -\frac{10}{5} = -2$

$$x = \pm \sqrt{-2} = \pm \sqrt{2} \sqrt{-1} = \boxed{\pm \sqrt{2} i}$$

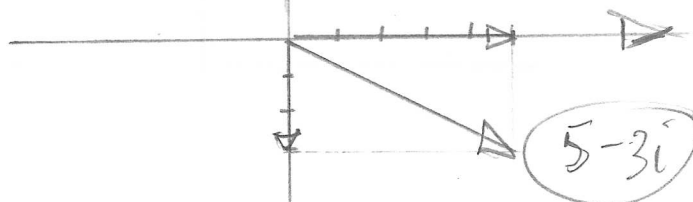
p149

1.- Representa gráficamente los siguientes n° complejos:

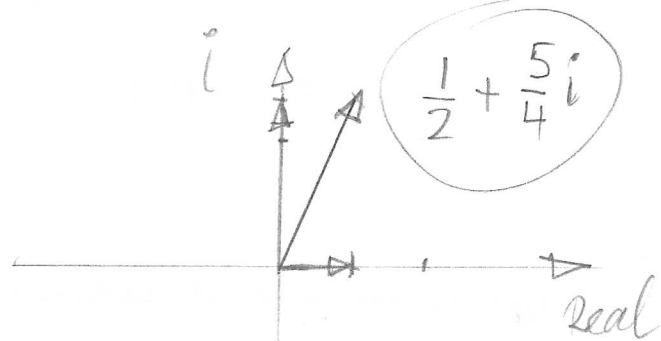
a) $5 + 3i$



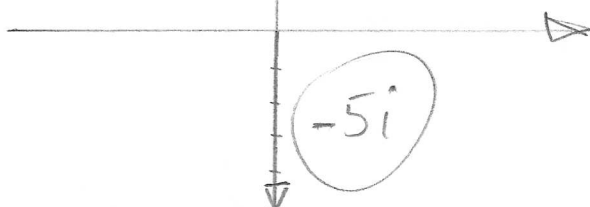
b) $5 - 3i$



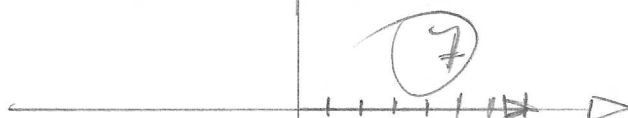
c) $\frac{1}{2} + \frac{5}{4}i$



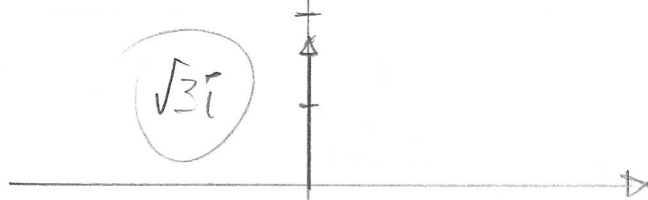
d) $-5i$



e) 7



f) $\sqrt{3}i$



g) 0



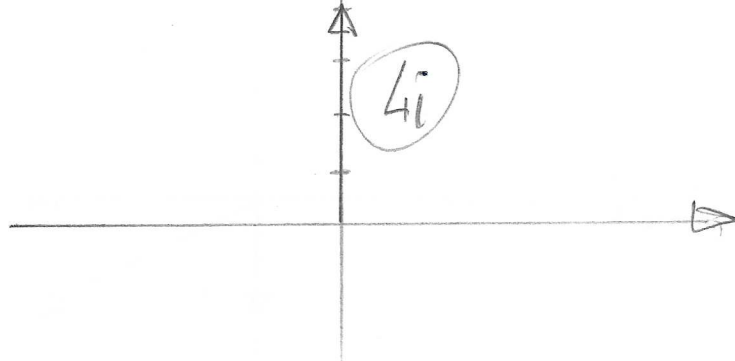
h) $-1-i$



i) -7



j) $4i$



3.- Expresa en forma trigonométrica los n.º complejos:

a) $z_1 = 1 + \sqrt{3}i$

$$|z_1| = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

$$\alpha = \arctg \frac{\sqrt{3}}{1} = \arctg \sqrt{3} = \frac{\pi}{3}$$

$$z_1 = 2 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

$$z_1 = 2 \angle \frac{\pi}{3}$$

b) $z_2 = \frac{1+i}{10} = \frac{1}{10} (1+i)$

$$|z_2| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\alpha = \arctg \frac{1}{1} = \arctg 1 = \frac{\pi}{4}$$

$$z_2 = \frac{\sqrt{2}}{10} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$

$$z_2 = \left(\frac{\sqrt{2}}{10} \right) \angle \frac{\pi}{4}$$

c) $z_3 = 10i$

$$|z_3| = \sqrt{0^2 + 10^2} = \sqrt{100} = 10$$

$$\alpha = \arctg \frac{10}{0} = \arctg \infty = \frac{\pi}{2}$$

$$z_3 = 10 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$$

$$z_3 = (10) \angle \frac{\pi}{2}$$

$$d) z_4 = -5$$

$$|z_4| = \sqrt{(-5)^2 + 0^2} = \sqrt{25} = \underline{\underline{5}}$$

$$\alpha = \operatorname{arctg} \frac{0}{-5} = \operatorname{arctg}(0) = \pi$$

$$z_4 = 5(\cos \pi + i \sin \pi)$$

$$z_4 = (5)\pi$$

4. Expresa en forma binómica el complejo:

$$z = \left(\frac{1 + \sin x + i \cos x}{1 + \sin x - i \cos x} \right)^n$$

[35] Demuestra la igualdad del paralelogramo:

$$|z+w|^2 + |z-w|^2 = 2(|z|^2 + |w|^2) \quad z, w \in \mathbb{C}$$

$$\begin{aligned} |z+w|^2 &= (z+w) \cdot (\bar{z} + \bar{w}) = z\bar{z} + \underbrace{z\bar{w} + w\bar{z}}_{2\operatorname{Re}(z\bar{w})} + w\bar{w} = \\ &= |z|^2 + |w|^2 + 2\operatorname{Re}(z\bar{w}) \end{aligned}$$

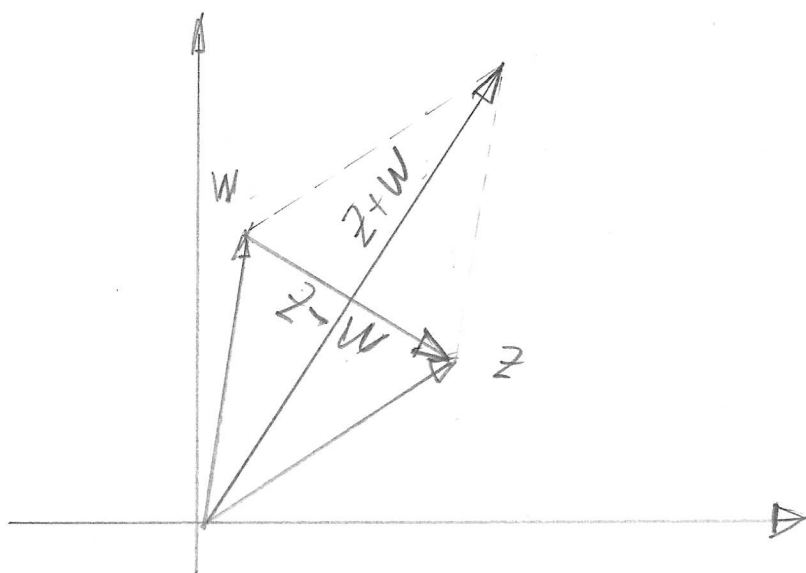
$$\begin{aligned} |z-w|^2 &= (z-w) \cdot (\bar{z} - \bar{w}) = z\bar{z} - z\bar{w} - \bar{z}w + w\bar{w} = \\ &= |z|^2 + |w|^2 - 2\operatorname{Re}(z\bar{w}) \end{aligned}$$

luego:

$$|z+w|^2 + |z-w|^2 = 2(|z|^2 + |w|^2)$$

su significado es:

La suma de los cuadrados de las longitudes de las diagonales de un paralelogramo es igual a la suma de los cuadrados de las longitudes de sus lados.



NOTA:

$$z\bar{z} = |z|^2 \quad ; \quad z\bar{w} = w\bar{z} \Rightarrow 2\operatorname{Re}(z\bar{w})$$

$$w\bar{w} = |w|^2$$

±, M.1. Práctica 1

2.- Halla los valores de z para los cuales $e^{3z} = 1$

Tomamos logaritmos:

$$\ln e^{3z} = \ln 1$$

$$3z = \ln 1$$

$$z = \frac{1}{3} \ln 1$$

$$z = \frac{1}{3} \left[\ln 1 + i(0 + 2k\pi) \right] = \frac{2k\pi}{3} i$$

$$k = 0, 1, 2, \dots$$

Cálculo del $\ln 1$

$$\ln 1 = \ln |1| + i \left(\begin{array}{c} \text{ } \\ 1+0i \end{array} \right)$$

$$r = \sqrt{1^2} = 1$$

$$\beta = \arctg\left(\frac{0}{1}\right) = \arctg(0) = 0 \quad \left. \vphantom{\arctg}\right\} \begin{array}{c} 1 \\ 0 \end{array}$$

b) $\ln(-4)$, $\ln i$, $\ln(1+i)$, $\log_{-1}(i)$

$\ln(-4)$

El módulo del complejo:

$$|w| = \sqrt{(-4)^2 + 0^2} = \sqrt{16} = 4$$

Su argumento:

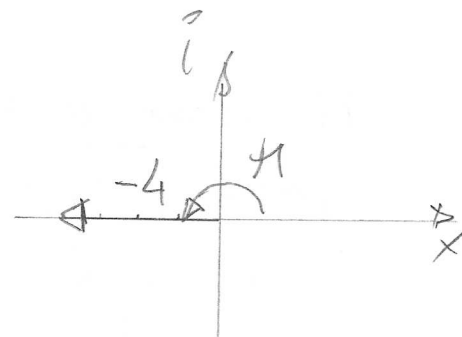
$$\beta = \operatorname{arctg} \frac{0}{-4} = \operatorname{arctg} 0 = \pi$$

luego:

$$\ln(w) = \ln|w| + i(\beta + 2k\pi) = \ln(4) + i(\pi + 2k\pi)$$

(tomando el valor principal $k=0$)

$$\boxed{\ln(-4) = \ln(4) + i\pi}$$



$\ln(i)$

El módulo del complejo:

$$|w| = \sqrt{0^2 + 1^2} = 1$$

Su argumento:

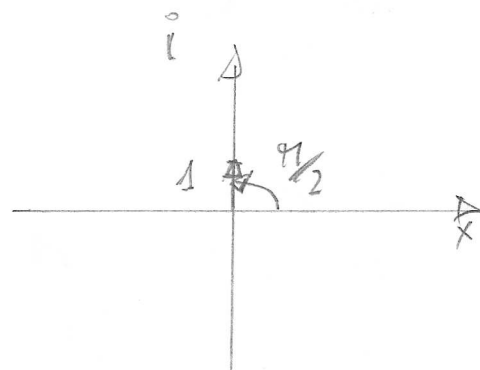
$$\beta = \operatorname{arctg} \frac{1}{0} = \operatorname{arctg} \infty = \frac{\pi}{2}$$

luego:

$$\ln(w) = \ln|w| + i(\beta + 2k\pi) = \ln|w| + i\beta$$

$$\ln(i) = \ln(1) + i\frac{\pi}{2} = i\frac{\pi}{2}$$

$$\boxed{\ln(i) = i\frac{\pi}{2}}$$



$$\ln(1+i)$$

El módulo del complejo:

$$|w| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

El argumento:

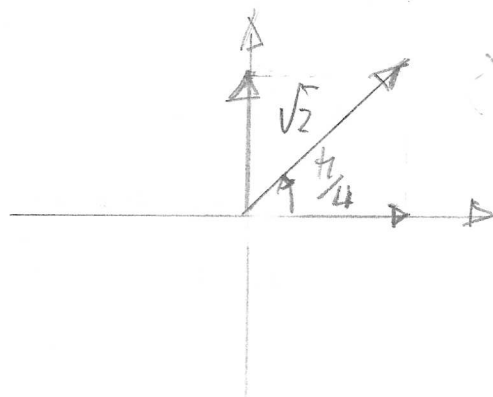
$$\beta = \arctg \frac{1}{1} = \arctg 1 = \frac{\pi}{4}$$

Luego:

$$\ln(w) = \ln|w| + i(\beta + 2k\pi) = \ln|w| + i\beta$$

VALOR PRINCIPAL

$$\ln(1+i) = \ln|\sqrt{2}| + i\frac{\pi}{4}$$



$$\log_{-1}(i) \leadsto \log_{(-1+0i)}(0+1i)$$

$$z = 0+1i$$

$$|z| = \sqrt{1^2} = \sqrt{1} = 1$$

$$\beta = \arctg\left(\frac{1}{0}\right) = \arctg(\infty) = \frac{\pi}{2}$$

$$b = -1+0i$$

$$|b| = \sqrt{1^2} = 1$$

$$\beta = \arctg\left(\frac{0}{-1}\right) = \arctg(0) = \pi$$

CAMBIO BASE

$$\log_b z = \frac{\ln z}{\ln b} = \frac{\ln|z| + i\beta}{\ln|b| + i\beta}$$

logaritmo principal

$$= \frac{\ln 1 + i\frac{\pi}{2}}{\ln 1 + i\pi} = \boxed{\frac{1}{2}}$$

TM1. Práctica 1

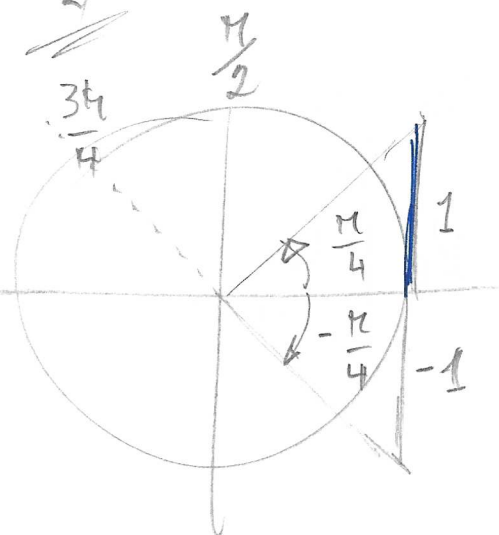
6.- Calcular la parte real de la parte principal del $\log_{1+i} (8-8i)$

$$z = 8-8i$$

$$8\sqrt{2} \left\{ \begin{aligned} |z| &= \sqrt{8^2 + 8^2} = \sqrt{64+64} = \sqrt{128} = 2^3\sqrt{2} = \underline{\underline{8\sqrt{2}}} \\ \beta &= \arctan\left(\frac{-8}{8}\right) = \arctan(-1) = -\frac{\pi}{4} \end{aligned} \right.$$

$$b = 1+i$$

$$\left\{ \begin{aligned} |b| &= \sqrt{1^2 + 1^2} = \sqrt{2} \\ \beta &= \arctan(1) = \frac{\pi}{4} \end{aligned} \right.$$



Definición de logaritmo:

$$\ln z = \ln |z| + i(\beta + 2k\pi)$$

El logaritmo principal: ($k=0$)

$$\boxed{\ln z = \ln |z| + i\beta}$$

Como el cambio de base logarítmica:

$$\boxed{\log_b z = \frac{\log_e z}{\log_e b} = \frac{\ln z}{\ln b}}$$

$$\log_{1+i} (8-8i) = \frac{\ln 8\sqrt{2} - \frac{\pi}{4}i}{\ln \sqrt{2} + \frac{\pi}{4}i} = \frac{(\ln 8\sqrt{2} - \frac{\pi}{4}i)(\ln \sqrt{2} - \frac{\pi}{4}i)}{(\ln \sqrt{2} + \frac{\pi}{4}i)(\ln \sqrt{2} - \frac{\pi}{4}i)} = \frac{(\ln 8\sqrt{2} - \frac{\pi}{4}i)(\ln \sqrt{2} - \frac{\pi}{4}i)}{(\ln \sqrt{2})^2 - (\frac{\pi}{4}i)^2}$$

$$\begin{aligned}
 & \ln 8\sqrt{2} \ln \sqrt{2} = \frac{\pi}{4} i \ln 8\sqrt{2} - \frac{\pi}{4} i \ln \sqrt{2} - \frac{\pi^2}{16} \\
 & = \frac{(\ln \sqrt{2})^2 + \frac{\pi^2}{16}}{\ln 8\sqrt{2} \ln \sqrt{2} - \frac{\pi^2}{16} - (\ln 8\sqrt{2} + \ln \sqrt{2}) \frac{\pi}{4} i} \\
 & = \frac{(\ln \sqrt{2})^2 + \frac{\pi^2}{16}}{(\ln \sqrt{2})^2 + \frac{\pi^2}{16}}
 \end{aligned}$$

Ans:

$$\operatorname{Re} \left[\log_{1+i} (8-8i) \right] = \frac{(\ln 8\sqrt{2} \ln \sqrt{2} - \frac{\pi^2}{16} - (\ln 8\sqrt{2} + \ln \sqrt{2}) \frac{\pi}{4} i)}{(\ln \sqrt{2})^2 + \frac{\pi^2}{16}}$$

0.303872

$$c) (-1)^i, i^i, 2^{1+i}, (1+i)^i, (-1+i)^{1/3}$$

$$(-1)^i$$

por definición:

$$z^w = e^{w \ln z}$$

O sea:

$$(-1)^i = e^{i \ln(-1)} = e^{i i \pi} = e^{-1} = \frac{1}{e}$$

$$(-1)^i = e^{-(i + 2k\pi)i}$$

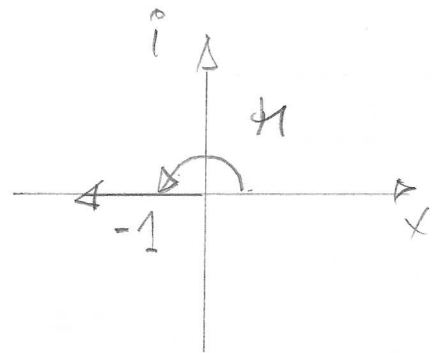
Cálculo de $\ln(-1)$

El módulo:

$$|z| = \sqrt{(-1)^2 + 0^2} = 1$$

El argumento:

$$\beta = \arctg \frac{0}{-1} = \arctg 0 = \pi$$



Como:

$$\ln z = \ln |z| + i \beta$$

$$\ln(-1) = \ln|1| + i\pi = i\pi$$

$$i^i$$

la definición potencia compleja:

$$z^w = e^{w \ln z}$$

Es decir:

$$i^i = e^{i \ln i} = e^{i i \frac{\pi}{2}} = e^{-\frac{\pi}{2}} = \frac{1}{e^{\frac{\pi}{2}}}$$

$$i^i = e^{-(\frac{\pi}{2} + 2k\pi)}$$

Cálculo de $\ln(i)$

El módulo del complejo:

$$|w| = \sqrt{0^2 + 1^2} = 1$$

El argumento:

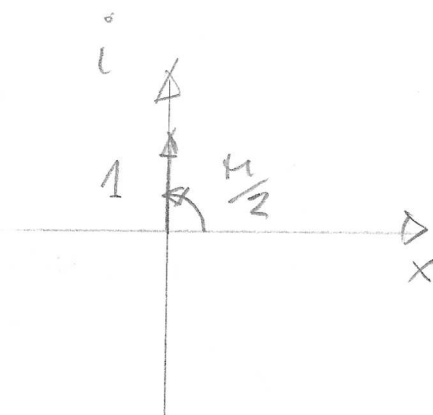
$$\beta = \arctg \frac{1}{0} = \arctg \infty = \frac{\pi}{2}$$

Luego:

$$\ln w = \ln |w| + i(\beta + 2k\pi) = \ln |w| + i\beta$$

$$\ln(i) = \ln 1 + i \frac{\pi}{2} = i \frac{\pi}{2}$$

Hecho anteriormente



2 $1+i$

Aplicando la def. de potencia compleja:

$$z^w = e^{w \ln z}$$
$$z^{1+i} = e^{(1+i) \ln z} = e^{\ln z + i \ln z}$$

$$= e^{(\ln z - 2k\pi) + i(\ln z + 2k\pi)}$$

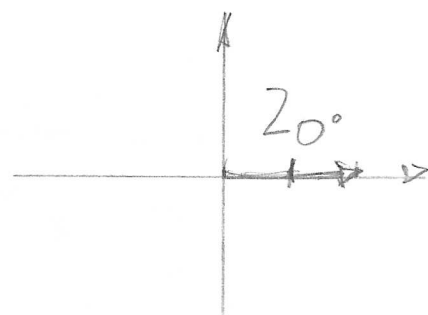
Da como solución

Cálculo de $\ln z$

$$|z| = \sqrt{2^2 + 0^2} = \sqrt{4} = 2$$

$$\beta = \arctg \frac{0}{2} = \arctg 0 = 0^\circ$$

$$\ln z = \ln |z| + i\beta = \ln 2$$



$$(1+i)^i$$

la definición de potencia: $z^w = e^{w \ln z}$

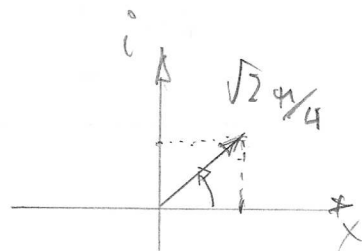
$$(1+i)^i = e^{i \ln(1+i)} = e^{i \left(\frac{\ln 2}{2} + i \frac{\pi}{4} \right)} = e^{-\frac{\pi}{4} + i \frac{\ln 2}{2}} = e^{-\left(\frac{\pi}{4} + 2k\pi\right) + i \frac{\ln 2}{2}}$$

Cálculo de $\ln(1+i)$

$$|z| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\beta = \operatorname{arctg} \frac{1}{1} = \operatorname{arctg} 1 = \frac{\pi}{4}$$

$$\ln z = \ln |z| + i \beta = \ln \sqrt{2} + i \frac{\pi}{4} = \frac{\ln 2}{2} + i \frac{\pi}{4}$$



$$(-1+i)^{1/3} = \sqrt[3]{-1+i}$$

$$\sqrt[n]{R \beta} \rightarrow r = \sqrt[n]{R}$$

$$\rightarrow \alpha = \frac{\beta + 2k\pi}{n}$$

Calculamos el radicando en forma polar:

$$z = -1 + i$$

$$|z| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$$

$$\beta = \operatorname{arctg} \frac{1}{-1} = \operatorname{arctg}(-1) = -\frac{\pi}{4} = \frac{3\pi}{4}$$

$$\sqrt{2} \frac{3\pi}{4}$$

Luego:

$$r = \sqrt[3]{\sqrt{2}} = \sqrt[6]{2}$$

$$\alpha = \frac{\frac{3\pi}{4} + 2k\pi}{3} = \frac{\pi}{4} + \frac{2k\pi}{3}$$

$$k=0, \quad \alpha_1 = \frac{\pi}{4}$$

$$k=1, \quad \alpha_2 = \frac{\pi}{4} + \frac{2\pi}{3} = \frac{3\pi + 8\pi}{12} = \frac{11\pi}{12}$$

$$k=2, \quad \alpha_3 = \frac{\pi}{4} + \frac{4\pi}{3} = \frac{3\pi + 16\pi}{12} = \frac{19\pi}{12}$$

$$\sqrt[6]{2} \frac{\pi}{4}, \quad \sqrt[6]{2} \frac{11\pi}{4}, \quad \sqrt[6]{2} \frac{19\pi}{12}$$

7. M. 1. Práctica 1

③ Halla la parte real de $(1-i)^{(1+i)}$

Llamamos z a la potencia

$$z = (1-i)^{(1+i)}$$

Por definición:

$$z^w = e^{w \ln z}$$

$$z = (1-i)^{(1+i)} = e^{(1+i) \ln(1-i)} \quad [1]$$

CÁLCULO DEL $\ln(1-i)$

$$(1-i) \begin{cases} r = \sqrt{1^2 + 1^2} = \sqrt{2} \\ \beta = \arctan\left(\frac{-1}{1}\right) = \arctan(-1) = -45^\circ = -\frac{\pi}{4} \end{cases}$$

$$\begin{aligned} \ln(1-i) &= \ln|1-i| + i(-\beta + 2k\pi) = \\ &= \boxed{\ln \sqrt{2} + i\left(-\frac{\pi}{4} + 2k\pi\right)} = \frac{\ln 2}{2} - \frac{\pi}{4} \end{aligned}$$

SUSTITUYENDO EN [1]

$$\begin{aligned} z &= (1-i)^{(1+i)} = e^{(1+i) \ln(1-i)} = \\ &= e^{(1+i) \left[\ln \sqrt{2} + i\left(-\frac{\pi}{4} + 2k\pi\right) \right]} = \\ &= e^{\ln \sqrt{2} + i\left(-\frac{\pi}{4} + 2k\pi\right) + i \ln \sqrt{2} - \left(-\frac{\pi}{4} + 2k\pi\right)} = \\ &= e^{\left(\ln \sqrt{2} + \frac{\pi}{4} - 2k\pi\right) + i\left(\ln \sqrt{2} - \frac{\pi}{4} + 2k\pi\right)} = \\ &= e^{\left(\ln \sqrt{2} + \frac{\pi}{4} - 2k\pi\right)} e^{i\left(\ln \sqrt{2} - \frac{\pi}{4} + 2k\pi\right)} \end{aligned}$$

7. EXPON.

$$= e^{(\ln \sqrt{2} + \frac{\pi}{4} - 2k\pi)} \left[\cos\left(\ln \sqrt{2} - \frac{\pi}{4} + 2k\pi\right) + i \sin\left(\ln \sqrt{2} - \frac{\pi}{4} + 2k\pi\right) \right]$$

↖ REAL ↗

$$\begin{aligned} \operatorname{Re} z &= e^{(\ln \sqrt{2} + \frac{\pi}{4} - 2k\pi)} \left[\cos\left(\ln \sqrt{2} - \frac{\pi}{4} + 2k\pi\right) \right] = \\ &= e^{(\ln \sqrt{2} + \frac{\pi}{4} - 2k\pi)} \left[\cos\left(\ln \sqrt{2} - \frac{\pi}{4}\right) \right] \\ &\quad \uparrow \\ &\quad ?? \end{aligned}$$

p161

8.- Interpretación gráfica de igualdades y desigualdades:
Representa los n° complejos z tales que:

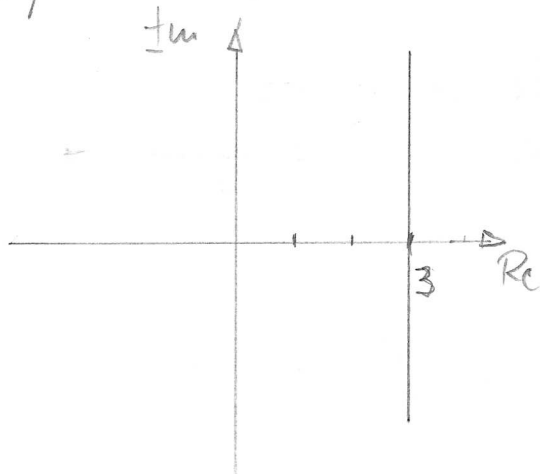
a) $\operatorname{Re}(z) = 3$

b) $\operatorname{Im}(z) = -1$

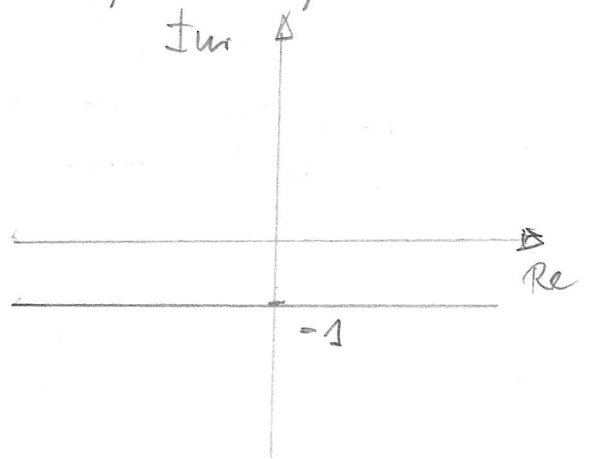
c) $-2 < \operatorname{Re}(z) < 3$

d) $1 < \operatorname{Im}(z) < 5$

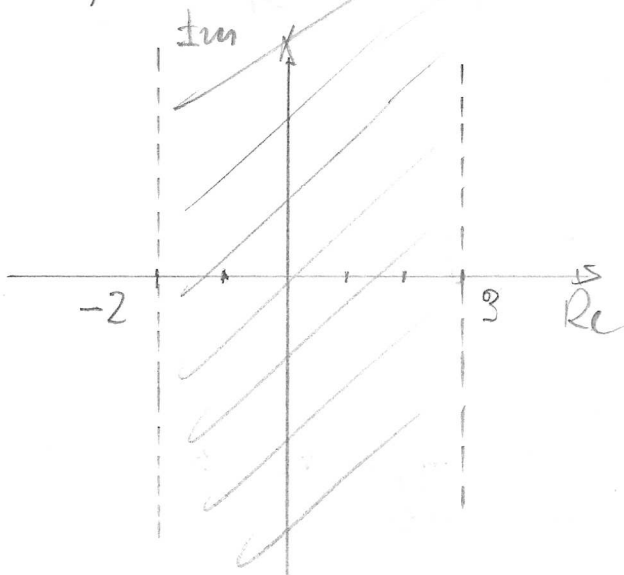
a) $\operatorname{Re}(z) = 3$



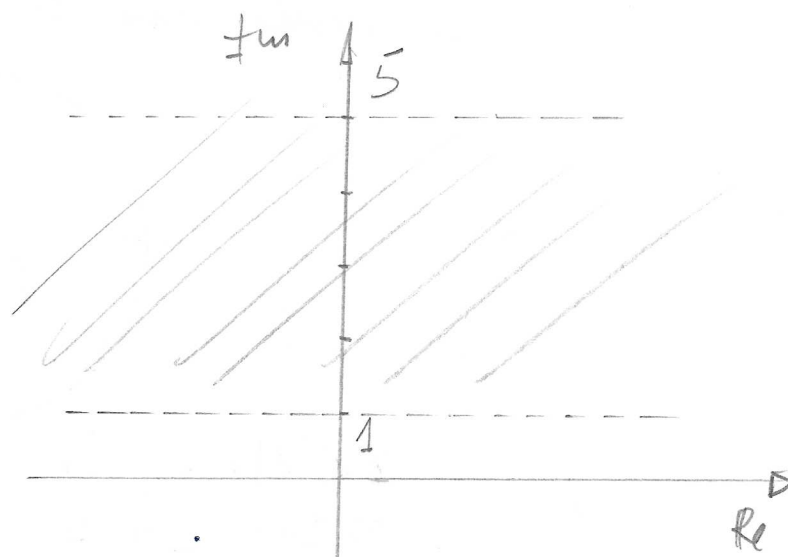
b) $\operatorname{Im}(z) = -1$



c) $-2 < \operatorname{Re}(z) < 3$



d) $1 < \operatorname{Im}(z) < 5$



9. Interpretación gráfica de igualdades y desigualdades

Representa los n.º complejos que verifican:

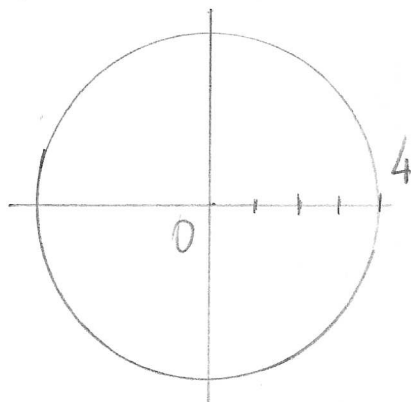
a) $|z| = 4$

b) $|z| < 4$

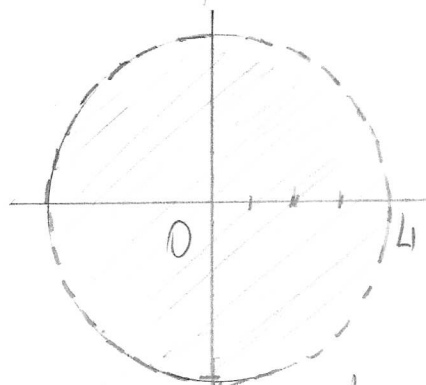
c) $2 < |z| < 4$

El módulo de un n.º complejo $z = a + bi$, es igual a la longitud del vector que lo representa, con origen $(0, 0)$ y extremo (a, b) .

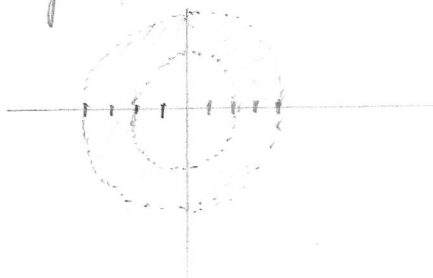
a) $|z| = 4$, los afijos estarán en una circunferencia con centro en O y radio 4



b) Si $|z| < 4$, los afijos estarán en el punto interior del círculo limitado por la circunferencia anterior



c) Si $2 < |z| < 4$, los afijos estarán entre las circunf. de radios 2 y 4, pero no en ellas



Ej. 48, p 164. ANAYA. BACHILLER 1º

Representa los n.º complejos z tales que $z + \bar{z} = -3$

$$z = x + iy$$

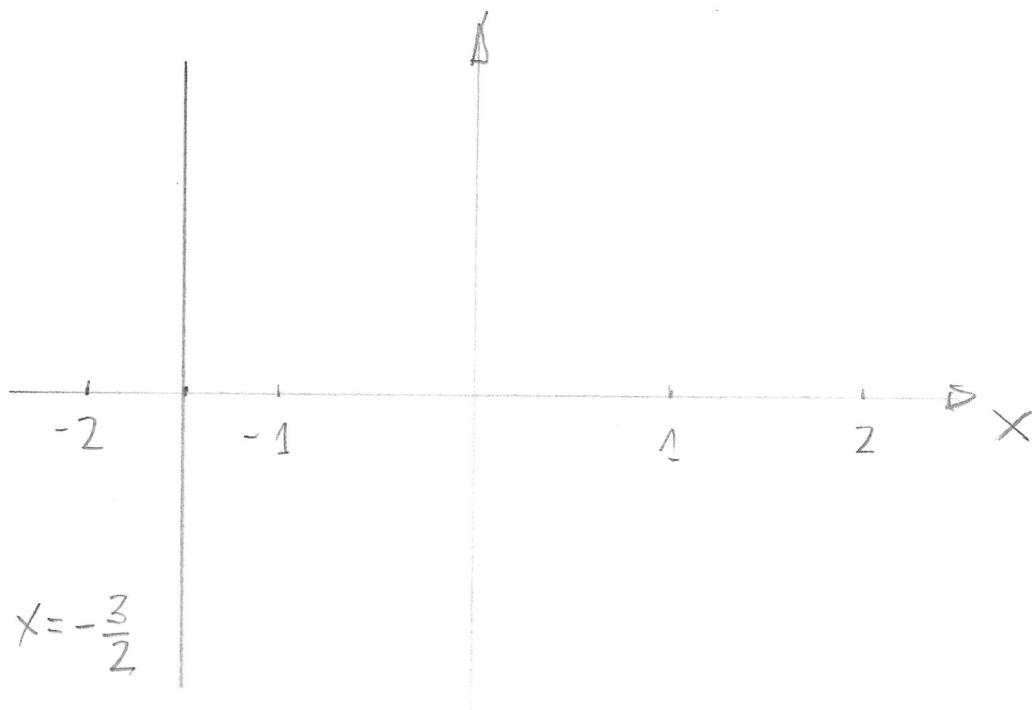
$$\bar{z} = x - iy$$

Sumando:

$$z + \bar{z} = -3$$

$$x + iy + x - iy = -3 ; 2x = -3 ; x = -\frac{3}{2}$$

La representación:



Ej. 49, p 164. ANAYA. BACHILLER 1º

Representa los n.º complejos que verifican:

a) $\bar{z} = -z$

b) $|z + \bar{z}| = 3$

c) $|z - \bar{z}| = 4$

Sean:

$$z = x + iy$$

$$\bar{z} = x - iy$$

a) $\bar{z} = -z$

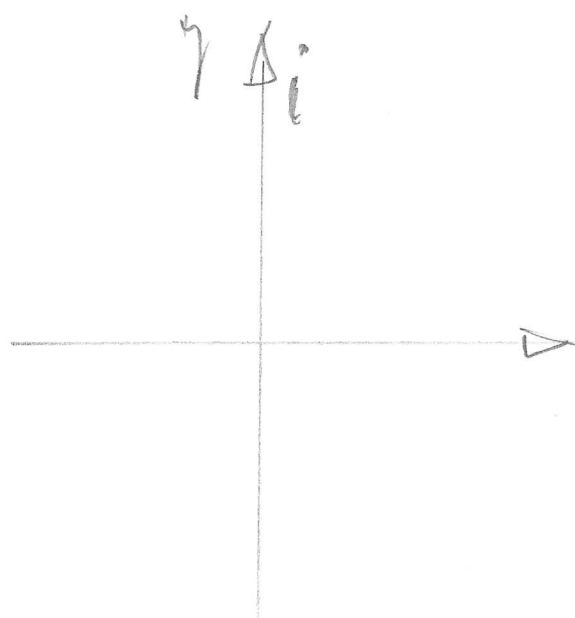
$$x - iy = -(x + iy)$$

$$x - iy = -x - iy$$

$$2x = 0$$

$$\boxed{x = 0}$$

$\Rightarrow$ EJE
IMAGINARIO



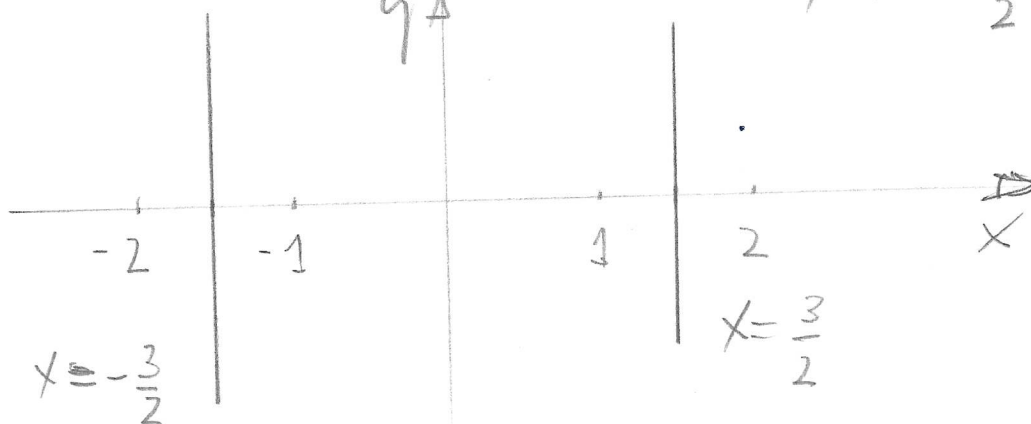
b) $|z + \bar{z}| = 3$

$$|x + iy + x - iy| = 3$$

$$|2x| = 3$$

$$2x = 3 ; x = \frac{3}{2}$$

$$2x = -3 ; x = -\frac{3}{2}$$



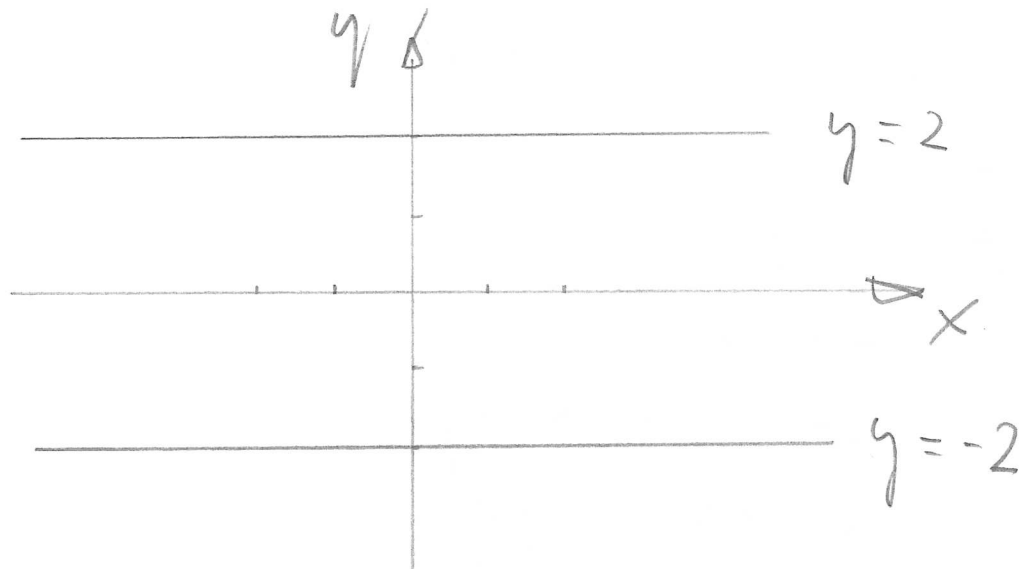
$$c) |z - \bar{z}| = 4$$

$$|\cancel{x} + iy - \cancel{x} + iy| = 4$$

$$|2yi| = 4$$

$$|2y| = 4$$

$$\begin{aligned} &\nearrow 2y = 4 ; y = \frac{4}{2} = 2 \\ &\searrow 2y = -4 ; y = -\frac{4}{2} = -2 \end{aligned}$$



Ej 50, p164. ANAYA. BACHILLER 1º

Escribe las condiciones que deben cumplir los n.º complejos cuya representación gráfica es la siguiente.

a) $\operatorname{Re}(z) = -3$

b) $\operatorname{Im}(z) = 2$

c) $-1 < \operatorname{Re}(z) < 1$

d) $0 \leq \operatorname{Im}(z) < 2$

e)
$$\begin{cases} -3 < \operatorname{Re}(z) < 2 \\ -2 < \operatorname{Im}(z) < 3 \end{cases}$$

f) $|z| = 3$

Ver dibujos
p164

PM1. Práctica 1

8.- Halla el lugar geométrico de los n.º complejos z , tales que:

$$\pm \operatorname{Im} \left(\frac{z+1}{z+2} \right) = \pm \operatorname{Im} z$$

Expresando $z = x + iy$

$$\pm \operatorname{Im} \left(\frac{x+iy+1}{x+iy+2} \right) = \pm \operatorname{Im} (x+iy)$$

$$\pm \operatorname{Im} \left(\frac{(x+iy+1)(x-iy+2)}{(x+iy+2)(x-iy+2)} \right) = \pm \operatorname{Im} (x+iy)$$

$$\pm \operatorname{Im} \left(\frac{x^2+3x+2+y^2+iy}{(x+2)^2+y^2} \right) = \pm \operatorname{Im} (x+iy)$$

$$\frac{y}{(x+2)^2+y^2} = y$$

$$\frac{1}{(x+2)^2+y^2} = 1$$

$(x+2)^2+y^2=1$, es decir, el lugar geométrico pedido es una circunferencia de centro $(-2, 0)$ y radio 1.